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Displacement field potentials for deformation in elastic Media: Theory and application to pressure-loaded boreholes

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ABSTRACT

This study demonstrates how analytical solutions for displacement field potentials of deformation in elastic media can be obtained from known vector field solutions for analog fluid flow problems. The theoretical basis is outlined and a geomechanical application is elaborated. In particular, closed-form solutions for deformation gradients in elastic media are found by transforming velocity field potentials of fluid flow problems, using similarity principles. Once an appropriate displacement gradient potential is identified, solutions for the principal displacements, elastic strains, stress magnitudes and stress trajectories can be computed. An application is included using the displacement gradient due to the internal pressure-loading of single and multiple wellbores. The analytical results give perfect matches with results obtained with an independent discrete element modeling method.

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1. Introduction

Mathematical descriptions of both fluid flow (including flow in porous media) and deformation in elastic media are possible with so-called complex potentials [1,2]. Such descriptions capture in the respective media, the spatial change of fluid velocity (fluid media) and elastic displacements (elastic media). The classical approach of complex analysis for fluid flow splits the *complex potential* in a *stream function* (imaginary part) and a *potential function* (real part). The *stream function* provides the velocity field and associated velocity gradient tensor for every fluid particle in every location. The *potential function* gives the pressure field everywhere in the flow studied. Countless flows can be described by specific stream function solutions [3–7], which all satisfy the Laplace equation. Fig. 1 shows an example [8] of flow past a cylinder with streamlines as velocity tangents obtained from a classical stream function (Fig. 1a) and pressure contours from the potential function (Fig. 1b). Time-of-flight contours given in Fig. 1a based on the analytical model closely match those independently modeled by a physical laboratory experiment [9,10] with marked fluid particles moving around a falling cylinder (Fig. 1c,d).

Unlike the analytical solutions, the physical laboratory experiment shows streamlines and isochrons affected by experimental “noise,” such as wall effects due to finite container size (Region A, Fig. 1c), variable adherence to the fluid boundary due some slip in places of un-intended lubrication (Region B, Fig. 1c), and even non-Newtonian effects, such as wider wake behind the cylinder (Region C, Fig. 1c), due to the high molecular weight of the cross-linked polymer fluid used [11–13]. In short, physical laboratory experiments and computational models, which include both analytical and discrete element solutions methods, are complementary. Analytical model descriptions give exact, closed-form solutions, but are often

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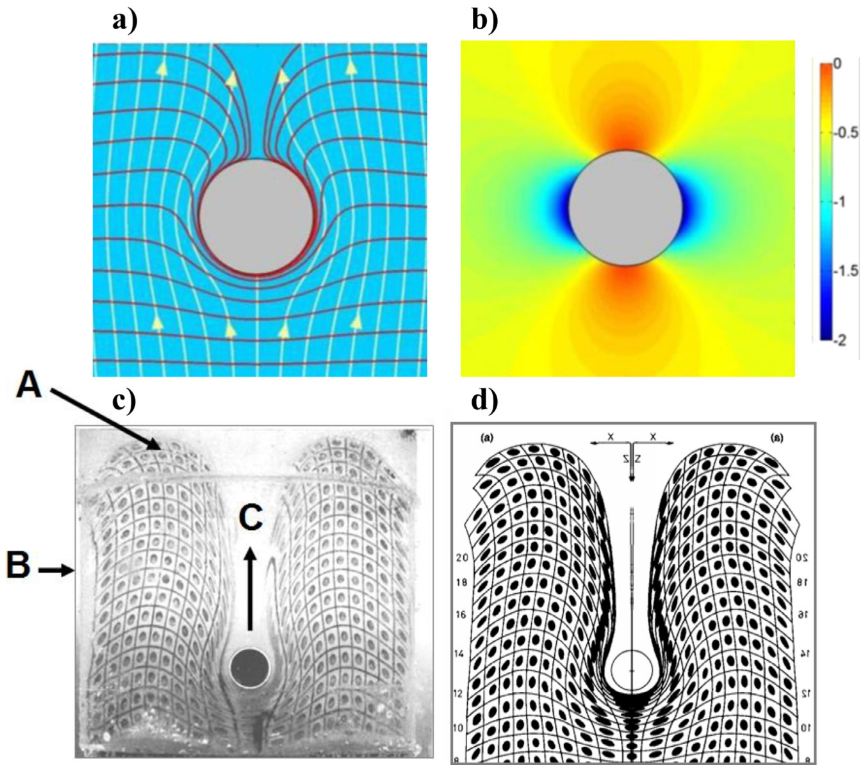

Fig. 1. a: Streamlines for non-inertial flow around a solid cylinder, marked with time-of-flight isochrons (red), based on analytical expressions for a uniform far-field flow and an anti-polar point doublet [8]. **b:** Corresponding pressure highs (red zones) and lows (blue zones). **c:** Laboratory experiment of a falling cylinder with embedded grid acting as streamline markers and time-of-flight contours [9]. Regions labeled A-C are explained in the text. **d:** Sketch of flow markers for the same falling cylinder [10]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

limited to isotropic material properties and simple boundary conditions. Advanced discrete element models can handle more complexity, but at expense of longer computation time unless coarse grid and mesh suffice, which give only approximate solutions.

When analytical solutions are available for flow problems they offer high resolution results (meshless, gridless) at low computation cost. For that reason a revival in stream function applications has been advocated in recent efforts to exploit the infinite resolution of closed-form solutions. Examples are flooding studies in hydrocarbon reservoirs [14,15], flow near hydraulically fractured wells [16], and fluid drainage near multi-fractured horizontal wells with fracture hits [17,18]. Separately, complex potentials and associated stress functions have been developed for elastic deformations to map the stress concentrations near internal boundaries [19–21]. Stress functions continue to provide closed-form solutions that can be applied to quantify the elastic response and possible failure of wellbores at high resolution, locating stress trajectories and neutral points of zero deviatoric stress [22,23].

Stream functions and stress functions can both be derived from complex potentials, and use similar tools of complex analysis. One may attempt a transformation of the stream function for a fluid flow system to a stress function to describe a geometrically similar elastic deformation. However, the actual transformation of a stream function to a stress function is less straightforward than alleged, and specific examples are rare if not completely absent in scholarly literature. Goodier [24] argued that by replacing viscosity with the shear modulus and strain rate with strain, the instantaneous, incompressible all-viscous and all-elastic problems are mathematically identical. In order to obtain valid similarity solutions for the kinematic (velocities /displacements) and dynamic (pressures) quantities of such moving fluids and deformed elastic media, the boundary conditions need to be similar (or similarly scalable). In fact, the procedure may be slightly more involved than simply replacing viscosity with shear modulus and strain rate with strain. In an elastic continuum, the deformation in response to a specific external force is instantaneous and results in a specific combination of finite strain and rigid body rotation (neglecting the option of volume change due to compressibility). Provided the finite strain is known everywhere, the stress scaling may require not only the shear modulus but also the Young modulus, because the deformation may involve both pure and simple shear motion. Incompressible, Newtonian fluids, display only shear motion and if scaled for a given (shear) viscosity and assuming steady-state flow, the strain-rate in every point remains time-independent (constant).

This study takes a more fundamental approach and shows how the velocity potential for a suitable flow problem can be manipulated to obtain valid solutions of the displacement potential for elastic problems. Closed-form solutions for advanced

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