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Public cooperation in two-layer networks with asymmetric interaction and learning environments

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ABSTRACT

Strategy updating is generally based on payoff comparison and strategy learning within the interaction pairs on networks in evolutionary games. In many previous works, the interaction and learning environments are assumed to be the same networks. However, in the real world, they might be different. In this work, we consider the spatial public goods game on two-layer networks, where the interaction and learning environments are represented by two asymmetric layers, respectively. We focus on the effects of edge overlap ω between the interaction and learning networks on the evolution of cooperation. The simulation results show that, the effects of ω on the evolution of cooperation depend on the synergy factor r. For relatively small r, higher overlap between the interaction and learning environments will be more favorable for cooperation. However, the situation is reverse for relatively large r, where the lower overlap between the interaction and learning environments results in higher level of cooperation. We also find that the asymmetry between the interaction and learning environments inhibits the coexistence of the cooperators and defectors. Furthermore, we show that the results of the model are robust to the underlying networks with different node degrees.

1. Introduction

Altruism and cooperation can be widely found in animal and human societies [1]. Understanding the spontaneous emergence and the maintenance of altruistic behaviors among selfish individuals confronted with social dilemmas, which seems incomprehensible in the context of Darwinian theory of evolution, remains an interesting problem and has attracted much attention. Evolutionary game theory provides a powerful theoretical framework for addressing this problem [2,3]. Over the past few decades, several typical game models, such as the prisoner's dilemma game (PDG) [4], the snowdrift game (SG) [5], and the public goods game (PGG) [6,7] are employed frequently to investigate the evolution of cooperation.

In a typical PGG played by N individuals [8], all individuals decide simultaneously to take one of the two pure strategies, cooperation (C) or defection (D). Every cooperator invests an amount c to the common pool, while defectors invest nothing. The total investment is multiplied by a synergy factor r ($r > 1$), and then is redistributed uniformly among all individuals irrespective of their contributions. It was shown that defectors will dominate the whole population for $1 < r < N$ in a

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well-mixed situation [9]. So far, various specific mechanisms in complex topologies have been proposed to understand the conundrum of cooperation in the context of the PGG, such as voluntary participation [8–10], punishment [11–14], reward [15], social diversity [16], mobility [17], tolerance [18,19], conformism [20], additive noise [21], investment heterogeneity [22], swarm intelligence [23] and income redistribution [24], and so on. Moreover, in recent years, multilayer networks instead of single-layer ones to describe complex population structures have been attracted increasing attention in evolutionary games [25–30] and others dynamics, including cascading failures [31–34], financial trading [35], competitive percolation [36–39], neuronal synchronization [40], epidemic spreading [41–43] and opinion dynamics [44], to name but a few.

Most previous studies of evolutionary games on graphs, with a few exceptions [45–48,51,52,54], implicitly assume that individuals update their strategies through choosing the role models from their interaction neighbours and then learning from them. It means that the interaction and learning environments are same. However, in real-life societies, individuals could learn not only from whom they interact with, but also from whom they do not interact with, such as parents and teachers. Hence, the situation in which the interaction and learning neighbourhoods are different has been considered. In Ref. [45], Ifti et al. first investigated the spatial continuous PDG when the neighborhoods of interaction and learning are different. They found that, when the neighbourhood sizes for interaction and learning differ by more than 0.5, cooperation is not sustainable. After that, related studies that considering the learning environment differs from the interaction environment have presented different conclusions in different models. In Ref. [46], Wu et al. studied the evolutionary PDG with two layered graphs (interaction layer and learning layer). They found that difference between the interaction and learning graphs can promote cooperation substantially. Ohtsuki et al. found that breaking the symmetry between interaction and learning networks undermines the evolution of cooperation [47,48]. Zhang et al. studied the effects of random partnerships on the evolution of cooperation, which are introduced to interaction or replacement networks. They showed that, compared with the case without random partnership, cooperation can be enhanced regardless of whether a random partnership is introduced to an interaction or replacement network [49]. Furthermore, Wang et al. considered two-layer scale-free networks with all possible combinations of degree mixing, wherein one network layer is used for the accumulation of payoffs and the other is used for strategy updating [50]. They found that breaking the symmetry through assortative mixing in one layer and/or disassortative mixing in the other layer, as well as preserving the symmetry by means of assortative mixing in both layers, impedes the evolution of cooperation. Moreover, Zhang et al. have investigated the implications of incongruence between the interaction network and the learning network for the evolution of cooperation in PDG and SG [51]. They found that cooperation will be severely inhibited if the learning network is very different from the interaction network. Besides, the influence of the different combinations of interaction and replacement networks with various intensity of selection on the evolution of cooperation has been explored by Suzuki et al. [52]. They found that the intensity of selection strongly affects the condition of the scale of interaction and reproduction in which the evolution of cooperation is strongly facilitated. In Ref. [53], Xia et al. presented a spatial PDG model to discuss the impact of separation between interaction neighborhood and learning neighborhood on the cooperative behaviors among players. They found that the medium-sized learning (interaction) neighborhood allows the cooperators to thrive and substantially favors the evolution of cooperation and the cooperation level can be greatly elevated when the interaction (learning) neighborhood is fixed. Recently, Tian et al. have further studied the impact of breaking the congruence of the interaction and learning networks on the evolution of cooperation in realistic social P2P environments [54]. The simulation results reveal that, for the separated structures of the neighborhoods, the interaction network has a critical effect on the evolution of cooperation and learning environments only have weaker impacts on the process.

Inspired by these studies, we aim to investigate the evolutionary PGG on multilayer networks by considering the interaction and learning environments are asymmetric, and focus on the effects of diverse extent of overlap between the interaction and learning networks on the system's cooperation level. As an intrinsic feature of multilayer networks, the presence of overlap can substantially influence the evolution of cooperation of the system. Most recently, Battiston et al. considered PGGs on a multiplex network and investigated the interplay between the overlap in the structure of the layers and the synergy factors in different layers [30]. They found that enhanced public cooperation emerges only when a significant edge overlap is combined with at least one layer being able to sustain some cooperation by means of a sufficiently high synergy factor. However, the effects of overlap between the interaction and learning networks on the evolution of cooperation in PGG have not yet been investigated. It is still an open question that what might happen if the learning environment differs from the interaction environment when evolutionary PGG is adopted as a metaphor for investigating cooperation. As we will show below, the effects of overlap between the interaction and learning environments on the evolution of cooperation depend on the synergy factor r. For relatively small r, higher overlap between the interaction and learning environments is more beneficial for cooperation. While it is reverse for relatively large r, and lower overlap between the interaction and learning environments can lead to higher level of cooperation.

In the rest of this paper, we will first describe our model in detail, then present the numerical simulation results and the corresponding discussions, and finally give the conclusions.

2. Model

We consider the evolutionary PGGs on a two-layer multiplex network (duplex), where the interaction and learning environments are asymmetric. One layer denotes the interaction environment and the other denotes the learning environment. The duplex network is described by the pair of binary adjacency matrices $\{A^{[1]}, A^{[2]}\}$. For each layer α, $A^{[\alpha]} \equiv \{a_{ij}^{[\alpha]}\}$, and