Advances in Mathematics 338 (2018) 549–600

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Electroid varieties and a compactification of the space of electrical networks

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A R T I C L E I N F O

Article history:
Received 15 October 2017
Received in revised form 3 August 2018
Accepted 28 August 2018
Available online xxxx
Communicated by Ezra Miller

MSC:
05C05
05C10
82B20

Keywords:
Electrical networks
Groves
Grassmannian
Electroids
Matchings

A B S T R A C T

We construct a compactification of the space of circular planar electrical networks studied by Curtis–Ingerman–Morrow [4] and Colin de Verdière–Gitler–Vertigan [3], using cactus networks. We embed this compactification as a linear slice of the totally nonnegative Grassmannian, and relate Kenyon and Wilson's grove measurements to Postnikov's boundary measurements. Intersections of the slice with the positroid stratification leads to a class of electroid varieties, indexed by matchings. The uncrossing partial order on matchings arising from electrical networks is shown to be dual to a subposet of affine Bruhat order. The analogues of matroids in this setting are certain distinguished collections of non-crossing partitions.

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[1] T.L. was supported by NSF grant DMS-1160726.

https://doi.org/10.1016/j.aim.2018.09.014

1. Introduction

1.1. Circular planar electrical networks and groves

A **circular planar electrical network** is a weighted undirected graph Γ embedded into a disk, with distinguished boundary vertices on the boundary of the disk. Each edge is thought of as a resistor, and the weight of the edge is the conductance of that resistor. The electrical properties of Γ are encoded in a response matrix

$$\Lambda(\Gamma) : \mathbb{R}^{\#\text{boundary vertices}} \to \mathbb{R}^{\#\text{boundary vertices}}$$

which sends a vector of voltages at the boundary vertices, to the vector of currents induced through the same vertices. Two electrical networks are electrically-equivalent if