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Set-theoretic solutions of the Yang–Baxter equation, braces and symmetric groups [☆]

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ABSTRACT

We involve simultaneously the theory of braided groups and the theory of braces to study set-theoretic solutions of the Yang–Baxter equation (YBE). We show the intimate relation between the notions of “a symmetric group”, in the sense of Takeuchi, i.e. “a braided involutive group”, and “a left brace”. We find new results on symmetric groups of finite multipermutation level and the corresponding braces. We introduce a new invariant of a symmetric group (G, r) , the *derived chain of ideals* of G , which gives a precise information about the recursive process of retraction of G . We prove that every symmetric group (G, r) of finite multipermutation level m is a solvable group of solvable length $\leq m$. To each set-theoretic solution (X, r) of YBE we associate two invariant sequences of involutive braided groups: (i) the sequence of its *derived symmetric groups*; (ii) the sequence of its *derived permutation groups*; and explore these for explicit descriptions of the recursive process of retraction of (X, r) . We find new criteria necessary and sufficient to claim that (X, r) is a multipermutation solution.

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1. Introduction

Let V be a vector space over a field k . It is well-known that the “Yang–Baxter equation” on a linear map $R : V \otimes V \rightarrow V \otimes V$, the equation

$$R_{12}R_{23}R_{12} = R_{23}R_{12}R_{23}$$

(where R_{ij} denotes R acting in the i, j place in $V \otimes V \otimes V$), gives rise to a linear representation of the braid group on tensor powers of V . When $R^2 = \text{id}$ one says that the solution is involutive, and in this case one has a representation of the symmetric group on tensor powers. A particularly nice class of solutions is provided by set-theoretic solutions, where X is a set and $r : X \times X \rightarrow X \times X$ obeys similar relations on $X \times X \times X$, [11]. Of course, each such solution extends linearly to $V = kX$ with matrices in this natural basis having only entries from 0,1 and many other nice properties.

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