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Set-theoretic solutions of the Yang–Baxter equation, braces and symmetric groups ☆

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ABSTRACT

We involve simultaneously the theory of braided groups and the theory of braces to study set-theoretic solutions of the Yang–Baxter equation (YBE). We show the intimate relation between the notions of *"a symmetric group"*, in the sense of Takeuchi, i.e. *"a braided involutive group"*, and *"a left brace"*. We find new results on symmetric groups of finite multipermutation level and the corresponding braces. We introduce a new invariant of a symmetric group (G, r), *the derived chain of ideals of* G, which gives a precise information about the recursive process of retraction of G. We prove that every symmetric group (G, r) of finite multipermutation level m is a solvable group of solvable length $\leq m$. To each set-theoretic solution (X, r) of YBE we associate two invariant sequences of involutive braided groups: (i) the sequence of its *derived symmetric groups*; (ii) the sequence of its *derived permutation groups*; and explore these for explicit descriptions of the recursive process of retraction of (X, r). We find new criteria necessary and sufficient to claim that (X, r) is a multipermutation solution.

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Contents

1. Introduction

Let V be a vector space over a field k. It is well-known that the “Yang–Baxter equation” on a linear map $R : V \otimes V \to V \otimes V$, the equation

$$R_{12}R_{23}R_{12} = R_{23}R_{12}R_{23}$$

(where R_{ij} denotes R acting in the i, j place in $V \otimes V \otimes V$), gives rise to a linear representation of the braid group on tensor powers of V. When $R^2 = \mathrm{id}$ one says that the solution is involutive, and in this case one has a representation of the symmetric group on tensor powers. A particularly nice class of solutions is provided by set-theoretic solutions, where X is a set and $r : X \times X \to X \times X$ obeys similar relations on $X \times X \times X$, [11]. Of course, each such solution extends linearly to $V = kX$ with matrices in this natural basis having only entries from 0,1 and many other nice properties.

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