Composite Structures 205 (2018) 1–10

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Length-scale effect in dynamic problems for thin biperiodically stiffened cylindrical shells

Barbara Tomczyk*, Anna Litawska

Department of Structural Mechanics, Lodz University of Technology, al. Politechniki 6, 90-924 Lodz, Poland

ARTICLE INFO

Keywords:
Biperiodically stiffened cylindrical shells
Mathematical modelling
Micro-vibrations

ABSTRACT

The objects of consideration are thin linearly elastic Kirchhoff-Love-type circular cylindrical shells having a periodically microheterogeneous structure in circumferential and axial directions (*biperiodic shells*). The aim of this contribution is to formulate and discuss a new mathematical averaged model for the analysis of selected dynamic problems for these shells. This, so-called, *general combined asymptotic-tolerance model* is derived by applying a certain extended version of the known tolerance (non-asymptotic) modelling procedure. This version is based on a new notion of *weakly slowly-varying functions*. Contrary to the starting exact shell equations with highly oscillating, non-continuous and periodic coefficients, governing equations of the proposed *combined model* have constant coefficients depending also on a cell size. Hence, this model can be applied to study the effect of a microstructure size on dynamic behaviour of the shells (*the length-scale effect*). An important advantage of this model is that *it makes it possible to analyse micro-dynamics of biperiodic shells independently of their macro-dynamics*. The differences between the proposed *general combined model* and the corresponding *known less accurate standard combined model* derived by means of the more restrictive concept of *slowly-varying functions* are discussed. As an example there are determined and analysed cell-depending micro-vibrations of the biperiodic shells under consideration.

1. Introduction

Thin linearly elastic Kirchhoff-Love-type circular cylindrical shells with a periodically micro-inhomogeneous structure in circumferential and axial directions are analysed. In the general case, by periodic inhomogeneity we shall mean here periodically variable shell thickness and periodically variable inertial and elastic properties of the shell material. Shells of this kind are termed *biperiodic*. Cylindrical shells with periodically spaced families of stiffeners as shown in Fig. 1 are typical example of such shells.

The dynamic problems of periodic shells are described by partial differential equations with highly oscillating, non-continuous, periodic coefficients. Hence, the direct application of these equations to investigations of engineering problems is noneffective even using computational methods. That is why there exists a number of various modelling methods leading to simplified averaged equations with constant coefficients. Periodic shells (plates) are usually described using *homogenized models* derived by means of *asymptotic methods*, cf. Lewiński and Telega [1], Andrianov et al. [2]. Unfortunately, in the models of this kind *the effect of a periodicity cell length dimensions* (called *the length-scale effect*) on the overall shell behaviour is neglected.

The length-scale effect can be taken into account using *the non-asymptotic tolerance averaging technique*, cf. Woźniak and Wierzbicki [3,4], Woźniak et al [5,6]. This technique is based on the concept of *tolerance relations* between points and real numbers related to the accuracy of the performed measurements and calculations. The tolerance relations are determined by *the tolerance parameters*. Contrary to starting equations of theories of microheterogeneous structures (partial differential equations with functional, highly oscillating, non-continuous coefficients), *governing equations of the tolerance models have coefficients which are constant or slowly-varying and depend on the period lengths of inhomogeneity*. Hence, these equations make it possible to analyse the length-scale effect.

Some applications of this method to the modelling of mechanical and thermomechanical problems for various periodic structures are shown in many works. The extended list of papers and books on this topic can be found in Woźniak and Wierzbicki [3], Woźniak et al. [5,6]. We mention here monograph by Tomczyk [7], where the length-scale effect in dynamics and stability of periodic cylindrical shells is investigated. In the last years the tolerance modelling was adopted for mechanical and thermomechanical problems of functionally graded structures, e.g. for heat conduction in longitudinally graded structures

* Corresponding author.
E-mail address: barbara.tomczyk@p.lodz.pl (B. Tomczyk).

https://doi.org/10.1016/j.compstruct.2018.08.090
Received 12 July 2018; Received in revised form 24 August 2018; Accepted 29 August 2018
Available online 05 September 2018

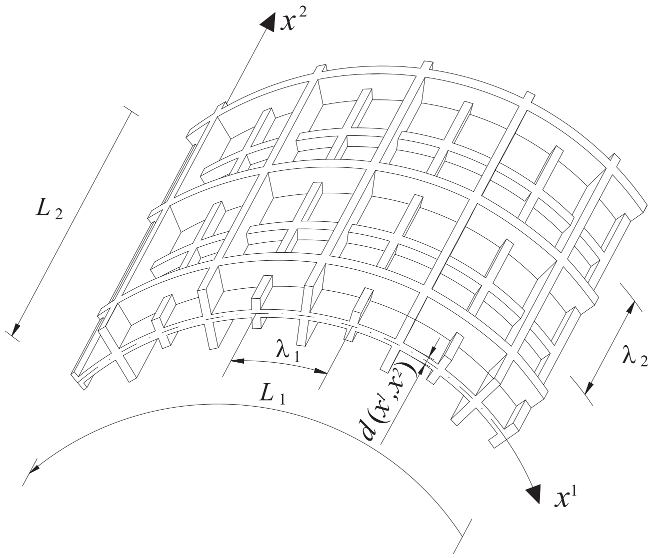

Fig. 1. A fragment of a shell with two families of biperiodically spaced ribs.

by Ostrowski and Michalak [8], for thermoelasticity of transversally graded laminates by Jędrysiak [9], Pazera and Jędrysiak [10], for dynamics of skeletonal functionally graded shallow shells by Michalak and Woźniak [11], for vibrations of annular plates with longitudinally graded structure by Wirowski [12], for vibrations of functionally graded thin plates by Kaźmierczak and Jędrysiak [13], for dynamics and stability of functionally graded cylindrical shells by Tomczyk and Szczerba [14–16].

A certain extended version of the tolerance modelling technique has been proposed by Tomczyk and Woźniak in [17]. This version is based on a new notion of *weakly slowly-varying functions* which is a certain extension of the well known concept of *slowly-varying functions*, cf. [3–6]. New *mathematical general tolerance models* of dynamic problems for thin shells with either one- or two-directional periodic microstructure in directions tangent to the shell midsurface, derived by means of the concept of *weakly slowly-varying functions*, have been proposed by Tomczyk and Litawska in [18,19]. The models mentioned above are certain generalizations of the corresponding *standard tolerance models* proposed in [7], which have been obtained by using the classical concept of *slowly-varying functions*. Note, that following [6] and [17], the concepts of *weakly slowly-varying and slowly-varying functions* are recalled in Section 3 of this paper.

The main aim of this contribution is to formulate and discuss a new averaged *general combined asymptotic-tolerance model* for the analysis of selected dynamic problems for *the biperiodic shells* under consideration. The model will be derived by applying *the combined modelling* which includes two techniques: *the consistent asymptotic modelling procedure* given by Woźniak [6] and the *extended tolerance non-asymptotic modelling technique* proposed by Tomczyk and Woźniak [17]. In the general case, the mentioned above two modelling procedures are independent of each other. Here, they are conjugated with themselves. It means that they are combined together into a new modelling technique. Governing equations of *the combined model* have constant coefficients depending also on a cell size. An important advantage of this model is that *it makes it possible to separate the macroscopic description of a certain problem from its microscopic description*. The differences between *the general combined model* proposed here and the corresponding *known standard combined model* presented by Tomczyk in [7] and derived by means of the more restrictive notion of *slowly-varying functions* will be discussed.

The second aim of this contribution is to determine and discuss the cell-depending micro-vibrations of the biperiodic shells under consideration. The results obtained from both the general and the standard combined models will be compared.

2. Starting equations

We assume that x^1 and x^2 are coordinates parametrizing the shell midsurface M in circumferential and axial directions, respectively. We denote $\mathbf{x} \equiv (x^1, x^2) \in \Omega \equiv (0, L_1) \times (0, L_2)$, where L_1, L_2 are length dimensions of M, cf. Fig. 1. Let $O\bar{x}^1\bar{x}^2\bar{x}^3$ stand for a Cartesian orthogonal coordinate system in the physical space R^3 and denote $\bar{\mathbf{x}} \equiv (\bar{x}^1, \bar{x}^2, \bar{x}^3)$. A cylindrical shell midsurface M is given by $M \equiv \{\bar{\mathbf{x}} \in R^3\colon \bar{\mathbf{x}} = \bar{\mathbf{r}}(x^1, x^2), (x^1, x^2) \in \Omega\}$, where $\bar{\mathbf{r}}(\cdot)$ is the smooth function such that $\partial\bar{\mathbf{r}}/\partial x^1 \cdot \partial\bar{\mathbf{r}}/\partial x^2 = 0$, $\partial\bar{\mathbf{r}}/\partial x^1 \cdot \partial\bar{\mathbf{r}}/\partial x^1 = 1$, $\partial\bar{\mathbf{r}}/\partial x^2 \cdot \partial\bar{\mathbf{r}}/\partial x^2 = 1$. It means that on M the orthonormal parametrization is introduced. Sub- and superscripts $\alpha, \beta, \ldots$ run over 1,2 and are related to x^1, x^2, summation convention holds. Partial differentiation related to x^α is represented by ∂_α. Moreover, it is denoted $\partial_{\alpha\ldots\delta} \equiv \partial_\alpha \cdots \partial_\delta$. Let $a^{\alpha\beta}$ stand for the midsurface first metric tensor. The time coordinate is denoted by $t \in \mathrm{I} = [t_0, t_1]$. Let $d(\mathbf{x})$, r stand for the shell thickness and the midsurface curvature radius, respectively.

Let λ_1 and λ_2 be the period lengths of the shell structure respectively in x^1- and x^2-directions. *The basic cell* Δ and an arbitrary cell $\Delta(\mathbf{x})$ with the centre at point $\mathbf{x} \in \Omega_\Delta$ are defined by means of: $\Delta \equiv [-\lambda_1/2, \lambda_1/2] \times [-\lambda_2/2, \lambda_2/2] \subset \Omega$, $\Delta(\mathbf{x}) \equiv \mathbf{x} + \Delta$, $\mathbf{x} \in \Omega_\Delta$, $\Omega_\Delta \equiv \{\mathbf{x} \in \Omega\colon \Delta(\mathbf{x}) \subset \Omega_\Delta\}$. The diameter $\lambda \equiv [(\lambda_1)^2 + (\lambda_2)^2]^{1/2}$ of Δ is assumed to satisfy conditions: $\lambda/d_{max} \gg 1$, $\lambda/r \ll 1$ and $\lambda/\min(L_1, L_2) \ll 1$. Hence, the diameter will be called *the microstructure length parameter*.

Setting $\mathbf{z} \equiv (z^1, z^2) \in [-\lambda_1/2, \lambda_1/2] \times [-\lambda_2/2, \lambda_2/2]$, we assume that the cell Δ has two symmetry axes: for $z^1 = 0$ and $z^2 = 0$. It is also assumed that inside the cell not only the geometrical but also elastic and inertial properties of the shell are described by symmetric (i.e. even) functions of $\mathbf{z} \equiv (z^1, z^2)$.

Denote by $u_\alpha = u_\alpha(\mathbf{x}, t)$, $w = w(\mathbf{x}, t)$, $\mathbf{x} \in \Omega$, $t \in \mathrm{I}$, the shell displacements in directions tangent and normal to M, respectively. Elastic properties of the shell are described by shell stiffness tensors $D^{\alpha\beta\gamma\delta}(\mathbf{x})$, $B^{\alpha\beta\gamma\delta}(\mathbf{x})$. Let $\mu(\mathbf{x})$ stand for a shell mass density per midsurface unit area. The external forces will be neglected.

The considerations are based on the well-known Kirchhoff-Love theory of thin elastic shells, cf. Kaliski [20].

It is assumed that the behaviour of the shell under consideration is described by the action functional

$$A(u_\alpha, w) = \int_0^{L_1}\int_0^{L_2}\int_{t_0}^{t_1} L(\cdot, \partial_\beta u_\alpha, \dot{u}_\alpha, \partial_{\alpha\beta} w, w, \dot{w})dt,\, dx^2 dx^1 \tag{1}$$

where lagrangian L is a highly oscillating function with respect to $\mathbf{x} \equiv (x^1, x^2) \in \Omega$ and has the well-known form, cf. [20]

$$L = -\frac{1}{2}(D^{\alpha\beta\gamma\delta}\partial_\beta u_\alpha \partial_\delta u_\gamma + 2r^{-1}D^{\alpha\beta 11} w \partial_\beta u_\alpha + r^{-2}D^{1111}ww + B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} w - \mu a^{\alpha\beta}\dot{u}_\alpha \dot{u}_\beta - \mu\dot{w}^2). \tag{2}$$

Under assumption that $\partial L/\partial(\partial_\beta u_\alpha)$ and $\partial L/\partial(\partial_{\alpha\beta} w)$ are continuous, from the principle of stationary action applied to $A(u_\alpha, w)$, we obtain the following system of Euler-Lagrange equations

$$\partial_\beta \frac{\partial L}{\partial(\partial_\beta u_\alpha)} + \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{u}_\alpha} = 0,$$

$$-\partial_{\alpha\beta}\frac{\partial L}{\partial(\partial_{\alpha\beta} w)} - \frac{\partial L}{\partial w} + \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{w}} = 0. \tag{3}$$

Combining (3) with (2) we arrive finally at the explicit form of *the fundamental equations of the shell theory under consideration*

$$\partial_\beta(D^{\alpha\beta\gamma\delta}\partial_\delta u_\gamma) + r^{-1}\partial_\beta(D^{\alpha\beta 11}w) - \mu a^{\alpha\beta}\ddot{u}_\beta = 0,$$

$$r^{-1}D^{\alpha\beta 11}\partial_\beta u_\alpha + \partial_{\alpha\beta}(B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} w) + r^{-2}D^{1111}w + \mu\ddot{w} = 0. \tag{4}$$

It can be observed that equations (4) coincide with the well-known governing equations of Kirchhoff-Love theory of thin elastic shells, cf. [20]. For periodic shells, coefficients $D^{\alpha\beta\gamma\delta}(\mathbf{x})$, $B^{\alpha\beta\gamma\delta}(\mathbf{x})$, $\mu(\mathbf{x})$ of