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Recurrent neurodynamic model of neuron with variable activation characteristic

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ABSTRACT

The article deals with a recurrent neurodynamic model of the neuron with a broad class of activation functions, including sigmoidal, stepwise, bounded linear and other ones. We consider the activation characteristic of the neuron, which is the dependence of the output signal on the input one. We demonstrate analytically that regardless of the choice of a particular activation function from the class of functions under consideration, the activation characteristic will have a characteristic property. When the value of the so-called modulating parameter is varied, the type of this characteristic will change from a function close to the selected activation function to a curve containing hysteresis loops. In other words, through the modulating parameter, the neuron model can be tuned to operate in monostable mode or one of the admissible multi-stability modes. We demonstrate how to determine the bifurcation points of the regime change, the corresponding boundaries of the multi-stability regions, and the conditions for finding solutions in them. The results of numerical experiments confirm the validity of the conclusions.

Introduction

One of the main tasks in the field of neural network modeling is to search for and develop new data processing methods as one of the ways to solve the problem of intelligent autonomous systems (Tiumentsev, 2017). Such systems should have the properties of high autonomy and adaptability to provide full or partial replacement of the human operator in various control and information processing systems, for example in monitoring systems based on the use of flying, ground-surface, and underwater unmanned vehicles.

Existing neurophysiological studies of working memory show (Egorov, Hamam, Fransén, Hasselmo, & Alonso, 2002; Egorov, Unsicker, Bohlen, & Halbach, 2006; Winograd, Destexhe, & Sanchez-Vives, 2008), that brain neurons can demonstrate persistent graded activity. Some authors have proposed the appropriate biologically based models of neurons (Goldman, Levine, Major, Tank, & Seung, 2003; Teramae & Fukai, 2005; Fransén, Tahvildari, Egorov, Hasselmo, & Alonso, 2006) with this property. These results were achieved due to the application of the hysteresis effect (Krasnosel'skii & Pokrovskii, 2012), which ensures that the models have the required multi-stability property.

In this regard, some authors have attempted to use hysteresis models of neurons in the implementation of artificial neural networks. In particular, the article (Tsuboshita & Okamoto, 2009; Okamoto, 2011) demonstrates how the representation of patterns in the form of continuous attractors in contrast to the conventional approach with discrete attractors can be formed in a network made up of this type of neurons. This method has successfully solved the problem topic-dependent document retrieval, surpassing the quality of some of the existing methods. Also, in Jin'no (2009) was solved 2-colorable graph coloring problem, using the hysteresis model of the neuron, as well as in Prostov and Tiumentsev (2015a) a neural network model of the finite state machine was built. We should note that attempts to apply hysteresis neurons have been made earlier, for example, in Yanai and Sawada (1990), where the authors used this kind of neurons in the associative Hopfield network. However, as was shown later in Tsuboshita and Okada (2010), some conclusions about the advantages of the resulting network were wrong.

As part of the problem of constructing a neural network model of context-dependent pattern recognition (Prostov & Tiumentsev, 2013) we also proposed a model of hysteresis neuron, discussed in detail in Prostov and Tiumentsev (2015b). In this paper, we showed that a simple dynamic model of a recurrent neuron could demonstrate both mono-stability and bi-stability, as well as tri-stability. In this case, we can control the transition from one state to another through a global modulating parameter. Besides, we demonstrated how the obtained

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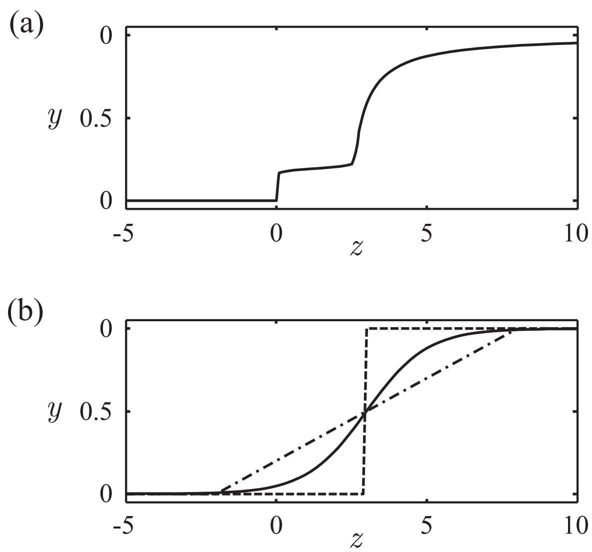

Fig. 1. The activation functions of the *S*-class: (a) considered in (Prostov & Tiumentsev, 2015b); (b) sigmoidal (solid line), stepwise (dashed line) and a limited linear (dash-dotted line).

properties of the model could be applied to increase the robustness of the network, as well as to identify temporal correlations between the recognized patterns. However, we considered in Prostov and Tiumentsev (2015b) the model of neuron with a specific activation function, which is shown in Fig. 1a. In this regard, we attempt to generalize the model of neuron proposed earlier by considering a whole family of activation functions, as well as highlighting the general properties that the model will have when using a particular activation function.

Model of the neuron

The following system of equations determines the formal model of a neuron considered in this paper:

$$\begin{cases} du/dt = \alpha y + i(\mathbf{w},\mathbf{x}) - \mu u, \\ y = s(h(u,\theta)), \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is the potential of the neuron (internal state), $y \in [0;1]$ is the frequency of the neuron (output signal), $\mathbf{w} \in \mathbb{R}^M$ is the weighting vector, $\mathbf{x} \in \mathbb{R}^M$ is the input pattern (input signal), $\alpha \in [0;+\infty)$ is the weight coefficient of the recurrent connection, μ is the dissipation constant characterizing the rate of decrease of the potential, and $\theta \in (0;+\infty)$ is the modulating parameter of the neuron, changing its *activation characteristic*, i.e. the dependence of the output signal on the input signal. Next, we show that the parameters α, μ and θ are interrelated, so, for certainty, we assume that the values of the first two parameters are selected and stated in advance, and the value of the last one is variable.

We introduce the function of *external excitation* $i: \mathbb{R}^M \times \mathbb{R}^M \to \mathbb{R}$ to generalize the method of transforming the input signal. In the particular case, this function can be, for example, a scalar product of vectors or some other vector measure. Next, for brevity, we will denote it by the letter i, omitting the arguments, but implying their presence.

The function of *modulation* $h: \mathbb{R} \times (0;+\infty) \to \mathbb{R}$ transforms the potential of the neuron u in accordance with the value of modulating parameter θ as follows:

$$h(u,\theta) = u/\theta. \tag{2}$$

The function of *activation* $s: \mathbb{R} \to [0;1]$ transforms the modulated potential of the neuron u to the output value y. As we said earlier, we will consider a whole class of activation functions, called the *S*-class, i.e. $s \in S$. The *S*-class itself is defined as the set of nondecreasing bounded piecewise smooth functions defined on the whole set of real numbers:

$$S = \{s\colon \inf s = 0,\ \sup s = 1,\ \forall\, a, b \in \mathbb{R}\colon a \leqslant b \Rightarrow s(a) \leqslant s(b),\ s \in \mathbb{C}^1(\mathbb{R}\backslash E) \quad \text{where} \quad E \subset \mathbb{R}\colon \forall\, e \in E\ \ \exists \lim_{x\to\pm e} s'(x)\} \tag{3}$$

For certainty, we denote the set of discontinuity points of the function itself as the set R, i.e. $s \in \mathbb{C}^0(\mathbb{R}\backslash R)$, and we assume that $\forall\, r \in R\ s(r) = 0.5\cdot\lim_{x\to -r} s(x) + 0.5\cdot\lim_{x\to +r} s(x)$. Also note that the values of the lower and upper bounds (respectively, values 0 and 1) we choose for the convenience of working with the output variable y. These bounds do not limit the class of functions considered in the sense that any function which having other limit values, but satisfying the remaining conditions, can be transformed to the equivalent function of *S*-class by applying the appropriate scaling operator. Thus, as noted earlier, the functions common in the neural network models such as sigmoidal, stepwise, and bounded linear functions can be referred to the *S*-class of functions (see Fig. 1b).

Equilibrium points of the model

To find the stable equilibrium points of the neuron model, we simplify the system (1) by eliminating the explicit entry of the output variable y into it:

$$du/dt = \alpha\, s(h(u,\theta)) + i - \mu u. \tag{4}$$

We introduce the variable $z = u/\theta$ and expand the function $h(u,\theta)$, renaming the right side of the resulting equation as a function $F(z)$ and equating it to zero:

$$F(z) = \alpha\, s(z) + i - \mu\theta z = 0. \tag{5}$$

Then the roots of this equality will set the equilibrium points of the system (1). Based on the expression (2), the corresponding output values of the model will be determined by the relationship $y_i^* = s(z_i^*)$. The stability of points will be determined from the condition that for $F'(z_i^*) < 0$ the point will be stable, and for $F'(z_i^*) > 0$ it will be unstable. Accordingly, if at some point the derivative of the function $F(z)$ is zero or not defined, then additional studies are needed. However, we will not consider this issue in detail, since it is not the primary subject of this paper.

The subset of *S*-class functions, for which it is possible to obtain the solution to the equality (5) analytically, is very small. In general, we have to evaluate the solution and investigate its properties numerically or graphically. It will be more convenient to search for the solution if we rewrite the equality (5) as follows:

$$i = -\alpha\, s(z) + \mu\theta z, \tag{6}$$

which allows us to graphically depict the activation characteristic of the neuron, taking into account the dependence of the input variable $\mathbf{x}$ on the value i and the dependence of the output variable y on the value z.

For example, the graphical solutions of the (1) system in various planes in the case of using the sigmoid activation function are shown in Fig. 2: (a) under the condition $\alpha \leqslant 4\mu\theta$, when there exists a unique and stable solution y^*; (b) under the condition $\alpha > 4\mu\theta$, when there exists one unstable y_2^* and two stable solutions y_1^* and y_3^*. As we can see, depending on the values of the parameters, the neuron will perform two qualitatively different types of transformation of the external excitation i. In the first case (a) there is a continuous nonlinear mapping qualitatively corresponding to the properties of the sigmoid function. In the second case, (b) the map acquires a jump-like, trigger character at the points i^+ and i^-. Moreover, in the interval $i \in (i^-;i^+)$ the domain of existence of two solutions is formed, and the convergence to one of them depends on the initial conditions, i.e., in this area, there is a so-called hysteresis loop (Krasnosel'skii & Pokrovskii, 2012). Thus, according to bifurcation analysis, we have an assembly type catastrophe (Zeeman, 1977) concerning the parameters θ and i in the system. The bifurcation