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Short communication

On probabilistic ψ-contractions in Menger probabilistic metric spaces ☆

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Abstract

In this paper, we prove some equivalence for probabilistic ψ-contractions in Menger probabilistic metric spaces, by which we answer Open question 3 in Mihet and Zaharia (2016) [11] affirmatively.

Keywords: Fixed point; Probabilistic ψ-contraction; Menger probabilistic metric spaces

1. Introduction

There are several kinds of probabilistic contractions such as probabilistic φ-contractions [4,8], probabilistic $(\varphi, \varepsilon - \lambda)$-contractions [6,9], probabilistic (φ, b_n)-contractions [7,10] and probabilistic ψ-contractions [2,3] on Menger probabilistic metric spaces. Some fixed point theorems for these probabilistic contractions are obtained (see [11] and the references therein).

Definition 1.1 *([4,8]).* Let $(X, F, *)$ be a Menger probabilistic metric space. A mapping $T : X \to X$ is called a probabilistic φ-contraction if it satisfies the following condition:

$$F_{Tx,Ty}(\varphi(t)) \geq F_{x,y}(t), \forall x, y \in X, \forall t > 0,$$

where $\varphi : [0, \infty) \to [0, \infty)$ is a gauge function.

Denote Φ the class of gauge functions $\varphi : [0, \infty) \to [0, \infty)$ satisfying: $0 < \varphi(t) < t$ and $\lim_{n\to\infty} \varphi^n(t) = 0$ for all $t > 0$.

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And denote Φ_ω the class of gauge functions $\varphi : [0, \infty) \to [0, \infty)$ satisfying: for each $t > 0$ there exists $r \geq t$ such that $\lim_{n\to\infty} \varphi^n(r) = 0$.

Obviously, $\Phi \subset \Phi_\omega$ and the inclusion is strict.

For probabilistic φ-contractions, Jachymski in [8] obtained the following result:

Theorem 1.2 *([8]). Let $(X, F, *)$ be a complete Menger probabilistic metric space with $*$ of H-type. If T is a probabilistic φ-contraction with $\varphi \in \Phi$, then T is a Picard mapping.*

In 2015, Fang [4] obtained the following theorem as a generalization of Theorem 1.2.

Theorem 1.3 *([4]). Let $(X, F, *)$ be a complete Menger probabilistic metric space with $*$ of H-type. If T is a probabilistic φ-contraction with $\varphi \in \Phi_\omega$, then T is a Picard mapping.*

Definition 1.4 *([2,3]).* Let $(X, F, *)$ be a Menger probabilistic metric space. A mapping $T : X \to X$ is called a probabilistic ψ-contraction if it satisfies the following condition:

$$F_{Tx,Ty}(t) \geq F_{x,y}(\psi(t)), \forall x, y \in X, \forall t > 0.$$

where $\psi : [0, \infty) \to [0, \infty)$ is a gauge function.

Now we give three classes of gauge functions which will be used in the next.

Denote $\Psi = \{\psi : [0, \infty) \to [0, \infty) | \psi(t) > t, \lim_{n\to\infty} \psi^n(t) = \infty, \forall t > 0\}$.

$\Psi_1 = \{\psi : [0, \infty) \to [0, \infty) | \lim_{n\to\infty} \psi^n(t) = \infty, \forall t > 0\}$.

$\Psi_\omega = \{\psi : [0, \infty) \to [0, \infty) | \forall t > 0, \exists r \in (0, t]$ such that $\lim_{n\to\infty} \psi^n(r) = \infty$ and $\psi^{-1}(0) = 0.\}$

In 2016, Mihet and Zaharia [11] improved several fixed point results concerning probabilistic $(\varphi, \varepsilon - \lambda)$-contractions, probabilistic (φ, b_n)-contractions and probabilistic ψ-contractions. They obtained the following results:

Theorem 1.5 *([11]). Let $(X, F, *)$ be a complete Menger probabilistic metric space with $*$ of H-type and $\varphi : [0, \infty) \to [0, \infty)$ be a function with the properties: i) $\varphi((0, 1)) \subset (0, 1)$; ii) $\lim_{n\to\infty} \varphi^n(t) = 0$ for all $t > 0$. Then every probabilistic $(\varphi, \varepsilon - \lambda)$-contraction is a Picard mapping.*

Theorem 1.6 *([11]). Let $(X, F, *)$ be a complete Menger probabilistic metric space with $*$ of H-type and T be a probabilistic ψ_ω-contraction with $\psi_\omega \in \Psi_\omega$. Then T is a Picard mapping.*

Obviously, Theorem 1.6 is a generalization of Theorem 1.7.

Theorem 1.7 *([2,3]). Let $(X, F, *)$ be a complete Menger probabilistic metric space with $*$ of H-type and T be a probabilistic ψ-contraction with $\psi \in \Psi$. Then T is a Picard mapping.*

At the end of paper [11], Mihet and Zaharia raised several questions. Two of these questions are the followings:

Open question 1: Does Theorem 1.5 remain true when $\varphi \in \Phi_\omega$?

Open question 3: Is it possible for any probabilistic ψ_ω-contraction T with $\psi_\omega \in \Psi_\omega$, to find $\psi \in \Psi$ such that T is a probabilistic ψ-contraction? That is, are Theorem 1.6 and Theorem 1.7 equivalent?

Just recently, Gregori et al. [5] proved the following theorem which showed that Theorem 1.2 and Theorem 1.3 are equivalent (see also [1]).

Theorem 1.8. *Let $(X, F, *)$ be a Menger probabilistic metric space and $T : X \to X$ be a mapping. Then T is a φ-contraction for some $\varphi \in \Phi$ if and only if T is a φ_ω-contraction for some $\varphi_\omega \in \Phi_\omega$.*

That is, in the framework of Menger spaces the class of probabilistic φ_ω-contractions for $\varphi_\omega \in \Phi_\omega$ coincides with the class of probabilistic φ-contractions for some $\varphi \in \Phi$. Using the same techniques of [5], **Open question 1** can be