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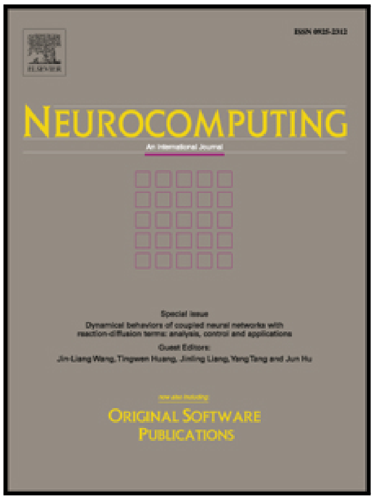

A unified deep artificial neural network approach to partial differential equations in complex geometries

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A unified deep artificial neural network approach to partial differential equations in complex geometries

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Abstract

In this paper we use deep feedforward artificial neural networks to approximate solutions to partial differential equations in complex geometries. We show how to modify the backpropagation algorithm to compute the partial derivatives of the network output with respect to the space variables which is needed to approximate the differential operator. The method is based on an ansatz for the solution which requires nothing but feedforward neural networks and an unconstrained gradient based optimization method such as gradient descent or a quasi-Newton method.

We show an example where classical mesh based methods cannot be used and neural networks can be seen as an attractive alternative. Finally, we highlight the benefits of deep compared to shallow neural networks and device some other convergence enhancing techniques.

Keywords: Deep neural networks, partial differential equations, advection, diffusion, complex geometries

1. Introduction

Partial differential equations (PDEs) are used to model a variety of phenomena in the natural sciences. Common to most of the PDEs encountered in practical applications is that they cannot be solved analytically but require various approximation techniques. Traditionally, mesh based methods such as finite elements (FEM), finite differences (FDM), or finite volumes (FVM), are the dominant techniques for obtaining approximate solutions. These techniques require that the computational domain of interest is discretized into a set of mesh points and the solution is approximated at the points of the mesh. The advantage of these methods is that they are very efficient for low-dimensional problems on regular geometries. The drawback is that for complicated geometries, meshing can be as difficult as the numerical solution of the PDE itself. Moreover, the solution is only computed at the mesh points and evaluation of the solution at any other point requires interpolation or some other reconstruction method.

In contrast, other methods do not require a mesh but a set of collocation points where the solution is approximated. The collocation points can be generated according to some distribution inside the domain of interest and examples include radial basis functions (RBF) and Monte Carlo methods (MCM). The advantage is that it is relatively easy to generate collocation points inside

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