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An inertial projection neural network for sparse signal reconstruction via l_{1-2} minimization[☆]

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ABSTRACT

In this paper, an inertial projection neural network (IPNN) is proposed for the reconstruction of sparse signals. Firstly, a nonconvex l_{1-2} minimization problem is presented for sparse signal reconstruction from highly coherent measurement matrices, instead of our familiar l_1 minimization which used standard convex relaxation. For solving this nonconvex l_{1-2} minimization problem, the IPNN is introduced. Under certain condition, the convergence of IPNN is proved. Finally, a series of experiments on various applications are conducted and experimental results show the effectiveness and performance of IPNN for the introduced l_{1-2} minimization method.

1. Introduction

Compressed sensing (CS) has been well developed over the last decade since the concept of which was formally proposed by Candès and Donoho [1,2] in 2006. As a new sampling theory, CS breaks through the bottleneck of shannon sampling theorem, which makes high-resolution signal acquisition become possible. We can directly acquire the essential information of a signal without the process of massive data acquisition. Nowadays, the sparse vector reconstruction problems have attracted much interest driven by important applications in signal processing [3], compressed sensing [4], machine learning [5] and statistics. A fundamental issue of CS is to recover an unknown sparse signal $\mathbf{x} \in \mathbb{R}^n$ from measurements $\mathbf{b} = \mathbf{Ax} + \mathbf{e}$, where $\mathbf{b} \in \mathbb{R}^m$, $\mathbf{A} \in \mathbb{R}^{m\times n} (m \ll n)$ is a sensing(measurements) matrix, and $\mathbf{e} \in \mathbb{R}^m$ represents a vector of measurement errors. It is a noise free problem in the case of $\mathbf{e} = \mathbf{0}$. The problem can be modeled as the following l_0-norm minimization problem [6–8]:

$$\min_{\mathbf{x}\in\mathbb{R}^n} \|\mathbf{x}\|_0 \text{ subject to } \mathbf{Ax} = \mathbf{b}, \tag{1}$$

where $\|\mathbf{x}\|_0$ is a discontinuous and nonconvex function, denoting the number of nonzero components of $\mathbf{x}$. In general, the problem (1) is NP-hard [9]. In order to overcome this difficulty, l_1 minimization or basis pursuit (BP) problem was proposed as an alternative

$$\min_{\mathbf{x}\in\mathbb{R}^n} \|\mathbf{x}\|_1 \text{ subject to } \mathbf{Ax} = \mathbf{b}, \tag{2}$$

where $\|\mathbf{x}\|_1 = \sum_{i=1}^n |x_i|$. Equivalence of problem (1) and problem (2) were given in [10] and [11]. Problem (2) is a convex optimization problem and can be transformed into a linear programming problem which is called Basis Pursuit (BP). At the same time, it can be solved by interior-point methods [12], gradient projection methods [13], homology methods [14] and iterative re-weighted least squares (IRLS) [15]. However, even though l_1 minimization is convex, it fails to find the sparest solution due to some leading entries (in magnitude) of $\mathbf{x}$ at times. In recent years, based on the fact that $\lim_{q\to 0} \|\mathbf{x}\|_q^q = \|\mathbf{x}\|_0$, l_q minimization problem [16–19] was proposed as a better approximation to l_0 minimization than l_1 minimization in the following:

$$\min_{\mathbf{x}\in\mathbb{R}^n} \|\mathbf{x}\|_q^q \text{ subject to } \mathbf{Ax} = \mathbf{b}, \tag{3}$$

where $\|\mathbf{x}\|_q$ represents l_q quasi-norm, defined by $\|\mathbf{x}\|_q = \left(\sum_{i=1}^n |x_i|^q\right)^{\frac{1}{q}}$. Beyond that, many other nonconvex penalties were proposed, such as smoothly clipped Absolute Deviation(SCAD) [20], Minimax concave penalty(MCP) [21], Log [22,23] and l_{1-2} norm [24]. From a perspective of sensing matrix, Gaussian and Bernoulli random matrices often have small coherence which

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is a way to characterize the dependence between columns of the matrix $\mathbf{A}$. However, sensing matrix with high coherence often inevitably arises in many applications such as direction-of-arrival (DOA) estimation [25], electroencephalography (EEG) source localization [26] and radar detection [27]. In these circumstances, the convex l_1 method leads to underperformance. In [24], Yin et al. proposed an l_{1-2} method, which outperforms l_1 and l_q method in dealing with such scenarios.

In recent decades, numerous neural networks for solving optimization problems have been extensively investigated [28,29]. Particularly, the recurrent neural network based on projective operator has been studied in the field of science and engineering, which can obtain real-time solution of optimization problems [30,31]. In [32], Xia and Wang presented a recurrent neural network for solving linear projection equations. Liu and Wang gave a one-layer projection neural network for solving non-smooth optimization problems with generalized convex objective functions in [33]. Based on a continuous-time projection neural network, Liu presented l_1 minimization algorithms for sparse signal reconstruction which were shown to be capable in [34].

In this paper we mainly discuss l_{1-2} minimization problem

$$\min_{\mathbf{x}\in\mathbb{R}^n} \|\mathbf{x}\|_1 - \|\mathbf{x}\|_2 \text{ subject to } \mathbf{Ax} = \mathbf{b}. \tag{4}$$

Obviously, this is a nonconvex optimization problem. Applying the knowledge of variable substitution and projective operator [35], we propose a novel neural network model for the minimization of the unconstrained l_{1-2} problem (5). With this approach, we can solve the problem in real-time and avoid the difficulty of the subgradient term. It is shown that the neural network is stable.

The rest of this paper is organized as follows. In Section 2, we present a neural network model and then discuss the existence and convergence of its solution. In Section 3, numerical experiments are conducted to illustrate the performance of the proposed neural network. Finally, we make the conclusion in Section 4.

2. Problem formulation

By means of the Lagrange multiplier, we convert the problem (4) into the unconstrained optimization problem

$$\min_{\mathbf{x}\in\mathbb{R}^n} \frac{1}{2}\|\mathbf{Ax}-\mathbf{b}\|_2^2 + \tau(\|\mathbf{x}\|_1 - \|\mathbf{x}\|_2). \tag{5}$$

There is no doubt that the objective of (4) is not differentiable. Instead of searching a method for minimizing the nonconvex objective function directly, we first transform the objective function. Suppose that in problem (5), the unknown variable $\mathbf{x}$ is replaced by $\mathbf{x} = \mathbf{u} - \mathbf{v}$, where $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ take all positive and negative elements of $\mathbf{x}$, respectively, i.e. $u_i = (x_i)_+,\ v_i = (-x_i)_+,\ (x_i)_+ = \max\{x_i, 0\}$, $i = 1, \dots, n$. With this replacement, it is easy to have that

$$\|\mathbf{x}\|_1 = \mathbf{1}_n^T(\mathbf{u}+\mathbf{v}) = \mathbf{1}_n^T\mathbf{u} + \mathbf{1}_n^T\mathbf{v},$$

$$\|\mathbf{x}\|_2 = \|\mathbf{u}-\mathbf{v}\|_2,$$

$$\mathbf{Ax} = \mathbf{A}(\mathbf{u}-\mathbf{v}),$$

where $\mathbf{1}_n = (1, \dots, 1)^T$. Therefore, the problem (5) can be re-written as

$$\begin{aligned}&\min_{\mathbf{u},\mathbf{v}\in\mathbb{R}^n} \frac{1}{2}\|\mathbf{A}(\mathbf{u}-\mathbf{v})-\mathbf{b}\|_2^2 - \tau\|\mathbf{u}-\mathbf{v}\|_2 + \tau\mathbf{1}_n^T\mathbf{u} + \tau\mathbf{1}_n^T\mathbf{v}\\ &\text{subject to } \mathbf{u} \geqslant 0, \mathbf{v} \geqslant 0\end{aligned} \tag{6}$$

here $\tau > 0$ is a regularization parameter. This problem is equivalent to the following nonconvex problem

$$\begin{aligned}&\min\ f(\mathbf{z}) = \frac{1}{2}\mathbf{z}^T\mathbf{Bz} - \tau\left(\mathbf{z}^T\mathbf{z}\right)^{\frac{1}{2}} + \mathbf{c}^T\mathbf{z}\\ &\text{subject to } \mathbf{z}\in S = \{\mathbf{z}\in\mathbb{R}^n | \mathbf{z} \geqslant 0\}\end{aligned} \tag{7}$$

where the objective function $f(\mathbf{z})$ is a nonconvex and continuous differential, and

$$\mathbf{z} = \begin{pmatrix}\mathbf{u}\\ \mathbf{v}\end{pmatrix},$$

$$\mathbf{B} = \begin{pmatrix}\mathbf{A}^T\mathbf{A} & -\mathbf{A}^T\mathbf{A}\\ -\mathbf{A}^T\mathbf{A} & \mathbf{A}^T\mathbf{A}\end{pmatrix},$$

$$\mathbf{c} = \tau\mathbf{1}_{2n} + \begin{pmatrix}-\mathbf{A}^T\mathbf{b}\\ \mathbf{A}^T\mathbf{b}\end{pmatrix}.$$

When $\mathbf{x}$, $\mathbf{A}$ are known, $\mathbf{b}$ can be calculated simply by $\mathbf{b} = \mathbf{Ax}$, then $\mathbf{z}$, $\mathbf{B}$ and $\mathbf{c}$ will be further obtained. From a perspective of sensing matrix $\mathbf{A}$, the properties of $\mathbf{A}$ are crucial for signal reconstruction in CS, thus the selection of A is what we are interested in. Both Restricted Isometry Properties (RIPs) and coherence are important tools to analyze the properties of sensing matrix. Gaussian or Bernoulli random matrices in general satisfy certain RIP, whereas they have low coherence. Owing to highly coherent measurement matrices are needed in many applications, it is necessary to consider l_{1-2} minimization, which has achieved excellent results in this case. The definition of the coherence as follows.

Definition 1. The coherence of a given matrix $\mathbf{A}$ is the largest absolute value of the cross-correlations between different columns from $\mathbf{A}$, namely,

$$\mu(\mathbf{A}) = \max_{i\neq j} \frac{\left|\mathbf{A}_i^T\mathbf{A}_j\right|}{\|\mathbf{A}_i\|_2\|\mathbf{A}_j\|_2},$$

where $\mathbf{A}_i$ is the i-th column vector of $\mathbf{A}$.

Remark 1. The coherence is a way to characterize the dependence between different columns from $\mathbf{A}$, and it is easy to compute. We describe a matrix $\mathbf{A}$ as a highly coherent measurement matrix if the coherence of which is large enough.

3. Neural network model and analysis

This section presents an IPNN for solving l_{1-2} minimization problem in CS. Based on scaled gradient projection, a neural network model is established to solve nonconvex problem (7). In addition, we demonstrate the convergence and the stability of the neural network.

3.1. Model description

Let $\mathbf{z}^* \in S$ as an optimal solution of (7), since $f(\mathbf{z})$ is twice differentiable, then $\mathbf{z}^* + t(\mathbf{z}-\mathbf{z}^*) \in S$ for all $t \in [0, 1]$ and $\mathbf{z} \in S$. Thus the function $q(t) = f(\mathbf{z}^* + t(\mathbf{z}-\mathbf{z}^*))$ is differentiable in (0, 1), and hence $q'(0) \geq 0$ since $q(t)$ reaches its minimum at $t = 0$. So

$$q'(0) = \nabla f(\mathbf{z}^*)^T(\mathbf{z}-\mathbf{z}^*) \geqslant 0, \forall\, \mathbf{z}\in S. \tag{8}$$

Finding an optimization of the (7) is equivalent to solve variational inequality(VI) (8), which can be treated like a natural framework of equilibrium problem in scientific and engineering fields. Therefore, our next task is to seek a appropriate approach for the VI (8).

In the light of [35], we have the following inertial projection neural network (IPNN) model