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Consensus of multi-agent systems with faults and mismatches under switched topologies using a delta operator method

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ABSTRACT

This paper studies the consensus of multi-agent systems with faults and mismatches under switched topologies using a delta operator method. Since faults and mismatches can result in failure of the consensus even for a fixed topology with a spanning tree, how to reach a consensus is a complicated and challenging problem under such circumstances especially when part topologies have no spanning tree. Although some works studied the influence of faults and mismatches on the consensus, there is little work on reaching a consensus for the multi-agent systems with faults and mismatches. In this paper, we introduce the delta operator to unify the consensus analysis for continuous, discrete, or sampled systems under one framework. We develop the theories on the delta operator systems first and then apply theories of the delta operator systems to the consensus problems. By converting the consensus problems into stability problems, we investigate and prove consensus and the associated conditions for systems 1) without any fault, 2) with a known fault, and 3) with unknown faults, under switching topologies with matching or mismatching coefficients. Numerical examples are provided and validate the effectiveness of the theoretical results.

1. Introduction

Collective behaviors widely exist in nature such as flocking of birds and schooling of fish. They have attracted more and more attention. Collective behaviors of groups rely on local interactions among individuals, which drives the emergence of coordination at the group scale [1]. Although the general mechanisms of coordination are not completely understood, known underlying mechanisms, for example, globally coordinated behaviors arise from local interactions, have helped us better understand the cooperative control of multi-agent systems [1,2]. Coordinations have received increasing interests for their broad applications [3–6].

The consensus, as one of the most fundamental collective behaviors, refers to reaching a common state for all initial conditions. This common state is not decided by centralized systems but by distributed systems with the local information of every agent and its neighbors [7,8]. The consensus problem has a long history, and it has received considerable attention in the last two decades [8–10]. The consensus problem under ideal conditions is well studied, and there is also limited work studying the influence of faults and mismatches on consensus. Reaching a consensus in the presence of faults and mismatches is very challenging as they can result in failure of the consensus even for a fixed topology with a spanning tree, particularly when the system is continuous, discrete or sampled.

Consensus with mismatching coefficients is of practical importance. Some preliminary results on the consensus are reported in [11–18]. According to the survey [11], consensus can be reached if the undirected network digraph is strongly connected. Because the undirected networks can be treated as special cases of directed networks where the adjacency matrices are symmetric, the conditions of directed networks are stricter than those of undirected networks. The article [12] reports that the first-order consensus can be achieved asymptotically if the union of the directed interaction graphs has a spanning tree sufficiently frequently as the system evolves. It is shown in [13] that the second-order consensus requires other conditions in addition to having a spanning tree. The work [17] states that a suitable Laplacian matrix and proper coupling strengths are indispensable to achieve a consensus for second-order systems under fixed topologies. That is, the second-order systems cannot reach a consensus with improper strength

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coefficients even for fixed topologies with a spanning tree. These studies and results indicate that achieving a consensus with mismatching coefficients is important and useful.

Mismatches resulting in failure of the consensus under fixed topologies does not necessary mean a failure under switched topologies. There is limited research on the consensus for topologies with mismatches and faults, considering varying topologies. In [15–18], conditions for the consensus are analysed with reference to the choice of coefficients under fixed topologies. But these papers did not explain when and how these topologies switch with the proper or improper coefficients.

With the development of cooperative control, multi-agent systems become increasingly complex, and faults cause severer impacts on system performance [19]. The sources of faults are multifarious, such as external measurements, internal settings and weighted deviations. These faults can cause system performance deterioration and lead to instability which can further result in catastrophic accidents. Thus, many effective fault tolerant control approaches have been proposed to improve system reliability [19–23]. The authors in [19] studied cooperative adaptive fuzzy tracking control for a network with unknown actuator faults which are limited to a weighted directed graph with a fixed topology. The paper [20] presents a cooperative actuator fault accommodation strategy for a team of multi-agent systems with a switching topology on the assumption that the digraph is always strongly connected. The paper [21] presents an adaptive fault-tolerant control scheme for leader-follower consensus control of uncertain mobile agents with actuator faults under a fixed topology. The study in [22] addresses the cooperative fault-tolerant tracking control problem for a class of multi-agent systems subject to mismatched parameter uncertainties, external disturbances and actuator faults under undirected connected graph with a fixed topology. The authors in [23] propose an H_∞ cooperative fault recovery control scheme confined to the fixed topology with a spanning tree. In actuality, a consensus is likely reached under switched topologies with unknown faults and a lack of the spanning tree. However, detailed realization is unknown yet.

With the wide use of computers in modern control engineering and the indispensable sampling operation to accomplish computerized processing, the sampled multi-agent systems have been studied in recent year [2,8,9]. This situation can also be found in the signal processing field [24,25]. These works separately study discrete or sampled multi-agent systems from continuous ones, which are effective for individual systems, but lack of generality. It is also hard to bridge and connect between continuous and sampled systems. Therefore, finding a suitable tool to unify the continuous and sampled multi-agent systems is very important. Based on the work on the delta operator [26–31], the delta operator is compatible with continuous and discrete time systems, and hence can be a good option for such a unifying tool.

This paper aims to study the consensus of multi-agent systems with faults and mismatches under switched topologies using a delta operator method, to address the major limitations of current research as mentioned above. The main contributions of this paper are as follows:

- We develop the method on the delta switched systems and this method unify consensus analysis for continuous, discrete and sampled systems under one framework;
- We investigate and prove the consensus and its associated conditions for systems with no fault, a known fault, and unknown faults originating from various sources; and
- We analyse consensus under switching topologies with matching and mismatching coefficients in every case, and demonstrate the associated conditions of consensus;

Our developed theorems in this paper have no special limitations. The topologies can be undirected or directed and the network can be balanced or unbalanced. The results in this paper can be applied, but not limited, to the following cases: no leader; switching leaders; switching between with and without a leader.

The rest of this paper are organized as follows: some basic concepts are provided in Section 2. Section 3 presents the main results for three cases of the multi-agent systems. Numerical examples are made in Section 4 to verify the theoretical analysis. Finally, the conclusion is given in Section 5.

Notations: X^T denotes the transpose of a matrix X, $P>(<)0$ denotes that the matrix P is symmetric and positive (negative) definite, I represents an identity matrix, C^1 represents the space of continuously differentiable functions, and $\mathbb{R}^N$ denotes the N-dimensional Euclidean space.

2. Preliminaries

In this section, some important basic graph theories are introduced.

A multi-agent system with n agents is considered in this paper. Basic graph theory is used to model the undirected or directed interaction among agents. The set of node indexes is $\mathcal{I}=\{1,2,\ldots,n\}$. Let a directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ describe the communication topology among the agents, where $\mathcal{V}=\{v_1,v_2,\ldots,v_n\}$, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$, and $\mathcal{A}=[a_{ij}]$. They are the set of nodes, a set of edges, and the corresponding adjacency matrix, respectively. We assume that there are no self loops in the networks. The set $\mathcal{N}_i=\{v_j\in\mathcal{V}:(v_i,v_j)\in\mathcal{E}, j\neq i, j\in\mathcal{I}, i\in\mathcal{I}\}$ stands for the set of neighboring node v_i. The matrix $L=[l_{ij}]$ is called as a Laplacian matrix, where 1) $l_{ij}=\sum_{k=1,k\neq i}^{n}a_{ik}$ for i=j; and 2) $l_{ij}=-a_{ij}$, for $i\neq j$, and $i,j\in\mathcal{I}$. A directed graph is called as a directed tree, when every node has exactly one parent except for the root. A directed tree is seen as a spanning tree of a directed graph when the tree connects all the nodes of the graph. More details can be found in [32]. A new matrix H is defined as $H=[h_{ij}]\in\mathbb{R}^{(n-1)\times(n-1)}$, where $h_{ij}=l_{ij}-l_{nj}$.

For the switched topologies, we assume that there are m topologies, where m is a positive constant. A finite set $\mathcal{J}=\{1,2,\ldots,m\}$ denotes all the switched topologies.

For the considered multi-agent system with n agents, every agent is modeled using the Gth-order model [17,33–35]:

$$\begin{aligned}
\dot{x}_i^{(0)}(t) &= x_i^{(1)}(t)\\
&\vdots\\
\dot{x}_i^{(G-2)}(t) &= x_i^{(G-1)}(t)\\
\dot{x}_i^{(G-1)}(t) &= u_i,\ i\in\mathcal{I}.
\end{aligned} \tag{1}$$

where $x_i(t)$ is the state of the ith agent, u_i is the control input, and $x_i^{(k)}$ stands for the kth derivative of x_i and $x_i^{(0)}=x_i$. Since it is harder to get higher order information of other agents in many circumstances, we consider the following consensus algorithm [33,34]:

$$\begin{aligned}
u_i(t) = {}& \beta_0\sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(x_j^{(0)}(t)-x_i^{(0)}(t))\\
&-\beta_1 x_i^{(1)}(t)-\cdots-\beta_{G-1}x_i^{(G-1)}(t),\ i=\mathcal{I},
\end{aligned} \tag{2}$$

where the positive constants $\beta_0,\beta_1,\beta_2,\ldots,\beta_{G-1}$ denote the (coupling or strength) coefficients, and a_{ij}, $i,j\in\mathcal{I}$, are the entries of the adjacency matrix $\mathcal{A}(\mathcal{G})$ for the given interaction topology $\mathcal{G}$. Let $\mathcal{K}$ denote the set of $\{0,\ldots,G-1\}$.

Remark 1. The methods and results in this paper are applicable to other consensus algorithms, such as the consensus algorithm on