Neurocomputing 000 (2018) 1–9

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Brief papers

Containment of linear multi-agent systems with disturbances generated by heterogeneous nonlinear exosystems

Xuxi Zhang*, Xianping Liu

College of Science, Harbin Engineering University, Harbin 150001, PR China

ARTICLE INFO

Article history:
Received 8 March 2018
Revised 27 June 2018
Accepted 29 June 2018
Available online xxx

Communicated by Xianming Zhang

Keywords:
Containment control
Multi-agent systems
Disturbance observer
Dynamic gain
Heterogeneous exosystems

ABSTRACT

This paper considers containment control problem of linear multi-agent systems with exogenous disturbances generated from heterogeneous nonlinear exosystems under directed networks. Based on dynamic gain technique, a nonlinear disturbance observer is proposed to compensate for the disturbances acting on the multi-agent systems. Then, distributed containment control law is presented, and sufficient conditions are given for solving the containment problem of the linear multi-agent systems. Finally, some numerical simulations are given to demonstrate the effectiveness of the proposed disturbance observers and control protocols.

1. Introduction

Recently, the cooperative control problem of multi-agent systems has received much attention due to its broad applications in many areas [1–3], such as unmanned air vehicles [4], flocking [5], sensor networks [6], rendezvous [7], formation control [8], and DC microgrids [9], etc. One of the fundamental problems in the area of cooperative control of multi-agent systems is the consensus problem. The main objective of consensus problem is to design distributed control law for each agent using local information from its neighbors such that all agents reach an agreement on certain quantities of interest.

Generally, in the light of the number of leaders in the multi-agent systems, the consensus algorithms can be roughly classified into three cases, i.e., leaderless consensus [10–13], consensus with one leader [14–17], and consensus with multiple leaders [18–27]. For the case of multiple leaders, the problem is also called containment control, and its target is to design control protocol for each agent such that the followers' states converge to the convex hull spanned by the leaders. Note that the containment control problem has many practical applications. For example, the containment control algorithm can guarantee that a group of robots reach their destination safely when only a few robots are equipped with sensors to detect the hazardous obstacle [28]. In [18], under a fixed undirected communication topology, a stop-go control policy was presented to drive a group of follower agents with single-integrator dynamics to the convex hull spanned by the leader agents. In [19], attitude containment control problem was discussed for multiple rigid bodies under undirected topology by using distributed finite time control method. In [20], containment control problem was investigated for first-order multi-agent systems under switching communication topologies. In [21,24], distributed containment control problem for single-integrator dynamics and double-integrator dynamics with stationary or dynamic multiple leaders were discussed under directed networks, respectively. In [22], necessary and sufficient criteria have been proposed for achieving containment control for first-order and second-order multi-agent systems, respectively. In [23], under random switching topologies, the containment control problem of second-order multi-agent systems was considered by using convex analysis and stochastic process. In practical applications, the dynamics of agents are often complicated, and may not be described by single-integrator or double-integrator dynamics. More recently, distributed containment control problem of multi-agent systems with general linear dynamics have also been investigated in [29–31].

It should be noted that, from a practical point of view, agents in a network are often influenced by external disturbances. Thus, it is desirable to discuss the external disturbances acting on the multi-agent systems, and some efforts have been made on this issue [32–35]. In [32], by using linear matrix inequalities method, a disturbance-observer-based consensus protocol was proposed for the consensus problem of second-order multi-agent

* Corresponding author.
E-mail addresses: zxx@hrbeu.edu.cn (X. Zhang), lxp@hrbeu.edu.cn (X. Liu).

https://doi.org/10.1016/j.neucom.2018.06.073
0925-2312/

systems with exogenous disturbances generated from some linear exosystems. In [33], under the assumption that the external disturbances can be generated by some nonlinear exosystems, a nonlinear disturbance-observer-based control law was given for the consensus of second-order multi-agent systems. In [34], the containment control problem of second-order multi-agent systems with external disturbances was studied by using an H_∞ approach. In [35], containment of linear multi-agent systems subject to exogenous disturbances, which were generated by some linear exosystems, was investigated under undirected topology by designing some linear disturbance observers. It is worth noting that, the advantage of the disturbance-observer-based control is that it can achieve asymptotical disturbance rejection, not just disturbance attenuation. On the other hand, a linear exosystem can only produce signals which are a combination of finitely many step functions and sinusoidal functions. The assumption that the disturbance is generated from some linear exosystems severely limits the applicability of the disturbance observer-based control approach. However, the containment problem of linear multi-agent systems with external disturbances has not been studied when the disturbances are generated by some nonlinear exosystems.

Motivated by the above analysis, in this paper, we investigate the distributed containment control problem for linear multi-agent systems with disturbances that are generated by some nonlinear exosystems. The main contributions are stated as follows: (1) Based on dynamic gain technique, a nonlinear disturbance-observer-based control law is proposed for solving the containment control problem of linear multi-agent systems with disturbances generated by some nonlinear exosystems. (2) The gain matrix of the disturbance observer is given by using the generalized inverse of matrix, and one sufficient condition is clearly given for the existence of the generalized inverse of matrix, which is shown in Remark 3.3. (3) The gain matrix and coupling gain of the distributed control are devised by solving algebraic Riccati equation and high-gain method, respectively.

The remainder of this paper is organized as follows. Section 2 introduces some useful preliminaries and the problem formulation. In Section 3, based on dynamic gain technique, containment control problem of linear multi-agent systems with external disturbances generated by some nonlinear exosystems is investigated by using a nonlinear disturbance-observer-based protocol. In Section 4, a numerical example is presented to illustrate the effectiveness of the proposed theoretical results. Finally, concluding remarks are given in Section 5.

2. Preliminaries and problem formulation

In this section, some basic concepts about algebraic graph theory are briefly introduced, which can be found in [36]. Then, the problem of containment control of linear multi-agent systems with disturbances generated by some nonlinear exosystems is formulated.

A digraph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ consists of a finite set of nodes $\mathcal{V}=\{1,2,\ldots,N\}$ and an edge set of ordered pairs of nodes $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. An edge of $\mathcal{G}$ from node i to node j is denoted by (i, j), representing that node j can get information from node i, but not necessarily vice versa. The set of neighbors of every node i is $\mathcal{N}_i=\{j:(j,i)\in\mathcal{E}\}$. The weighted adjacency matrix of a digraph $\mathcal{G}$ is a nonnegative matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$, where $a_{ij}>0\Leftrightarrow(j,i)\in\mathcal{E}$ and $a_{ii}=0$. Define the in-degree matrix $\mathcal{D}$ as a diagonal matrix $\mathcal{D}=\mathrm{diag}\{deg_{in}(i)\}$ with $deg_{in}(i)=\sum_{j\in N_i}a_{ij}$. Then, the Laplacian matrix of a weighted graph can be defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$. A directed path from node i_1 to node i_n is a sequence of ordered edges of the form $(i_k, i_{k+1}), k=1,\ldots,n-1$. A directed graph has a directed spanning tree if there exists at least one node called root node which has a directed path to all the other nodes. $\mathbf{1}_N$ is the $N\times 1$ vector with all entries being ones. An agent is called a leader if the agent has no neighbor, and a follower if the agent has at least one neighbor.

We consider a multi-agent system which is consisted of $M(M<N)$ followers and $N-M$ leaders. Suppose that the agents indexed by $1,2,\ldots,M$ are followers, and by $M+1,M+2,\ldots,N$ are leaders. And, the follower set and the leader set are denoted as $\mathscr{F}$ and $\mathscr{R}$, respectively. According to the fact that the leaders have no neighbor, the Laplacian matrix $\mathcal{L}$ associated with the digraph $\mathcal{G}$ can then be partitioned as

$$\mathcal{L}=\begin{bmatrix}\mathcal{L}_1 & \mathcal{L}_2\\ 0_{(N-M)\times M} & 0_{(N-M)\times(N-M)}\end{bmatrix} \tag{1}$$

where $\mathcal{L}_1\in\mathbb{R}^{M\times M}$ and $\mathcal{L}_2\in\mathbb{R}^{M\times(N-M)}$.

The dynamics of the ith follower is given by

$$\dot{x}_i=Ax_i+B(u_i+d_i),\ \ i\in\mathscr{F}, \tag{2}$$

where $x_i\in\mathbb{R}^n$, $u_i\in\mathbb{R}^m$ and $d_i\in\mathbb{R}^m$ are the state, control input and exogenous disturbance of the ith follower, respectively; $A\in\mathbb{R}^{n\times n}$ and $B\in\mathbb{R}^{n\times m}$ are constant system matrices of system (2), and we assume that (A, B) is a stabilizable pair. It is assumed that the disturbance d_i, $i\in\mathscr{F}$, is generated by the following nonlinear exosystem

$$\begin{aligned}\dot{\xi}_i&=E_i\xi_i+\phi_i(\xi_i),\\ d_i&=F_i\xi_i,\ \ i\in\mathscr{F},\end{aligned} \tag{3}$$

where $\xi_i\in\mathbb{R}^{q_i}$ is the state of the nonlinear exosystem (3), E_i and F_i are constant system matrices of the nonlinear exosystem (3), and the matrix pair (E_i, F_i) is observable. The function $\phi_i(\xi_i)$ in (3) is a globally Lipschitz function, i.e., there exists a positive constant $c_{\phi_i}>0$, such that

$$\|\phi_i(\xi_{i1})-\phi_i(\xi_{i2})\|\le c_{\phi_i}\|\xi_{i1}-\xi_{i2}\|, \tag{4}$$

for any $\xi_{i1},\xi_{i2}\in\mathbb{R}^{q_i}$.

The dynamics of the multiple leaders are represented as

$$\dot{x}_i=Ax_i,\ \ \ i\in\mathscr{R}, \tag{5}$$

where $x_i\in\mathbb{R}^n$ is the state of the ith leader.

Definition 2.1 [37]. A set $\mathcal{C}\in\mathbb{R}^n$ is said to be convex if $(1-\gamma)x+\gamma y\in\mathcal{C}$ whenever $x\in\mathcal{C}, y\in\mathcal{C}$ and $\gamma\in[0,\ 1]$. The convex hull of a finite set of points $X=\{x_1,\ldots,x_n\}$ in $\mathbb{R}^n$ is the minimal convex set containing all points $x_i, i=1,\ldots,n$, denoted by $Co\{X\}$. In particular, $Co\{X\}=\{\sum_{i=1}^n\gamma_i x_i|x_i\in X,\gamma_i\ge 0,\sum_{i=1}^n\gamma_i=1\}$.

Our aim is to design control protocols u_i depending on $x_i, x_j\in\mathcal{N}_i$ for (2) to guarantee that the states of the followers asymptotically converge to the convex hull spanned by those of the leaders.

The following assumption is used throughout the paper.

Assumption 2.2. The communication topology among the follower agents is directed. And, for each follower, there exists at least one leader that has a directed path to it.

Lemma 2.3 [19]. *If the interaction topology satisfies Assumption 2.2, then all the eigenvalues of $\mathcal{L}_1$ have positive real parts, each entry of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ is nonnegative, and each row of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ has a sum equal to one.*

3. Main results

In this section, we present a distributed control protocol for the general linear multi-agent systems (2) with exogenous disturbances generated by some nonlinear heterogeneous exosystems of the form (3).

In order to deal with the disturbance generated from (3), based on the dynamic gain technique, a nonlinear disturbance observer is proposed as

$$\dot{\eta}_i=(E_i-H_iBF_i)(\eta_i+H_ix_i)-H_i(Ax_i+Bu_i)$$