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Meta-kernelization using well-structured modulators[☆]

Eduard Eiben, Robert Ganian [*], Stefan Szeider

Algorithms and Complexity Group, TU Wien, Vienna, Austria

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ABSTRACT

Kernelization investigates exact preprocessing algorithms with performance guarantees. The most prevalent type of parameters used in kernelization is the solution size for optimization problems; however, also structural parameters have been successfully used to obtain polynomial kernels for a wide range of problems. Many of these parameters can be defined as the size of a smallest *modulator* of the given graph into a fixed graph class (i.e., a set of vertices whose deletion puts the graph into the graph class). Such parameters admit the construction of polynomial kernels even when the solution size is large or not applicable. This work follows up on the research on meta-kernelization frameworks in terms of structural parameters.

We develop a class of parameters which are based on a more general view on modulators: instead of size, the parameters employ a combination of rank-width and split decompositions to measure structure inside the modulator. This allows us to lift kernelization results from modulator-size to more general parameters, hence providing small kernels even in cases where previously developed approaches could not be applied. We show (i) how such large but well-structured modulators can be efficiently approximated, (ii) how they can be used to obtain polynomial kernels for graph problems expressible in Monadic Second Order logic, and (iii) how they support the extension of previous results in the area of structural meta-kernelization.

1. Introduction

Kernelization investigates exact preprocessing algorithms with performance guarantees. Similarly as in parameterized complexity analysis, in kernelization we study *parameterized problems*: decision problems where each instance I comes with a parameter k. A parameterized problem is said to admit a kernel of size $f : \mathbb{N} \to \mathbb{N}$ if every instance (I, k) can be reduced in polynomial time to an equivalent instance (called the *kernel*) whose size and parameter are bounded by $f(k)$. For practical as well as theoretical reasons, one is mainly interested in the existence of *polynomial kernels*, i.e., kernels whose size is polynomial in k. The study of kernelization has recently been one of the main areas of research in parameterized complexity, yielding many important new contributions to the theory.

The by far most prevalent type of parameter used in kernelization is the *solution size*. Indeed, the existence of polynomial kernels and the exact bounds on their sizes have been studied for a plethora of distinct problems under this parameter, and the rate of advancement achieved in this direction over the past 10 years has been staggering. Important findings were also obtained in the area of *meta-kernelization* [6,16,24], which is the study of general kernelization techniques and frameworks used to establish polynomial kernels for a wide range of distinct problems.

☆ A preliminary, shortened version of this paper appeared at IPEC 2015 (Eiben et al., [13]).

* Corresponding author.
E-mail address: rganian@ac.tuwien.ac.at (R. Ganian).

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In parameterized complexity analysis, an alternative to parameterization by solution size has traditionally been the use of *structural parameters*. But while parameters such as *treewidth* and the more general *rank-width* allow the design of FPT algorithms for a range of important problems, it is known that they cannot be used to obtain polynomial kernels for problems of interest [5]. Instead, the structural parameters used for kernelization often take the form of the size of minimum *modulators* (a modulator of a graph is a set of vertices whose deletion puts the graph into a fixed graph class). Examples of such parameters include the size of a minimum vertex cover [15,7] (a modulator into the class of edgeless graphs) or of a minimum feedback vertex set [8,22] (a modulator into the class of forests). While such structural parameters are not as universal as the structural parameters used in the context of fixed-parameter tractability, these results nonetheless admit an efficient preprocessing of instances where the solution size is large and for problems where solution size simply cannot be used (such as 3-coloring).

This paper follows up on the recent line of research which studies meta-kernelization in terms of structural parameters. Gajarský et al. [17] developed a meta-kernelization framework parameterized by the size of a modulator to the class of graphs of bounded treedepth on sparse graphs. Ganian et al. [19] independently developed a meta-kernelization framework using a different parameter based on rank-width and modular decompositions (see Section 2.4 for details). Our results build upon both of the aforementioned papers by fully subsuming the meta-kernelization framework of [19] and lifting the meta-kernelization framework of [17] to more general graph classes. The class of problems investigated in this paper are problems which can be expressed using *Monadic Second Order* (MSO) logic (see Section 2.5).

The parameters for our kernelization results are also based on modulators. However, instead of parameterizing by the *size* of the modulator, we instead measure the *structure* of the modulator through a combination of rank-width and split decompositions. Due to its technical nature, we postpone the definition of our parameter, the *well-structure number*, to Section 3; for now, let us roughly describe it as the number of sets one can partition a modulator into so that each set induces a graph with bounded rank-width and a simple neighborhood. We call modulators which satisfy our conditions *well-structured*. A less restricted variant of the well-structure number has recently been used to obtain meta-theorems for FPT algorithms on graphs of unbounded rank-width [14].

After formally introducing the parameter, in Section 4 we showcase its applications on the special case of generalizing the *vertex cover number* by considering well-structured modulators to edgeless graphs. While it is known that there exist MSO-definable problems which do not admit a polynomial kernel parameterized by the vertex cover number on general graphs, on graphs of bounded expansion this is no longer the case (as follows for instance from [17]).

1. On the class of graphs of bounded expansion, we prove that every MSO-definable problem admits a linear kernel parameterized by the well-structure number for edgeless graphs (Corollary 1).

In relation to the above, we also show in Theorems 1 and 2 that every MSO-definable problem admits a linear kernel parameterized by the well-structure number for the empty graph (without restriction on the expansion). We remark that these results represent a direct generalization of the meta-kernelization results in [19]. The proof is based on a combination of a refined version of the replacement techniques developed in [14] together with the annotation framework used in [19].

Before we can proceed to wider applications of our parameter in kernelization, it is first necessary to deal with the subproblem of finding a suitable well-structured modulator in polynomial time. We resolve this question for well-structured modulators to a vast range of graph classes:

2. We develop a 3-approximation algorithm for finding well-structured modulators to acyclic graphs (Section 5.1).
3. We develop constant-factor approximation algorithms for well-structured modulators to graph classes characterized by a finite set of forbidden induced subgraphs (Section 5.2).

Section 6 then contains our most general result, Theorem 5, which is the key for lifting kernelization results from modulators to well-structured modulators.

4. We prove that whenever a modulator to a graph class $\mathcal{H}$ can be used to obtain a polynomial kernel for some MSO-definable problem, this problem also admits a polynomial kernel when parameterized by the well-structure number for $\mathcal{H}$ as long as well-structured modulators to $\mathcal{H}$ can be approximated in polynomial time.

The remainder of Section 6 then deals with the applications of this theorem.

5. Since the class of graphs of treedepth bounded by some fixed integer can be characterized by a finite set of forbidden induced subgraphs, we use well-structured modulators to lift the results of [17] from modulators to well-structured modulators for all MSO-definable decision problems (Theorem 6).
6. Furthermore, by applying the *protrusion* machinery of [6,24] we show that, in the case of bounded degree graphs, parameterization by a modulator to acyclic graphs (i.e., a feedback vertex set) admits the computation of a linear kernel for all MSO-definable decision problems. By our framework it then follows that such modulators can also be lifted to well-structured modulators (Theorem 7).

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