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Energy-maximising control of wave energy converters using a moment-domain representation

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ABSTRACT

Wave Energy Converters (WECs) have to be controlled to ensure maximum energy extraction from waves while considering, at the same time, physical constraints on the motion of the real device and actuator characteristics. Since the control objective for WECs deviates significantly from the traditional reference "tracking" problem in classical control, the specification of an optimal control law, that optimises energy absorption under different sea-states, is non-trivial. Different approaches based on optimal control methodologies have been proposed for this energy-maximising objective, with considerable diversity on the optimisation formulation. Recently, a novel mathematical tool to compute the steady-state response of a system has been proposed: the *moment-based phasor transform*. This mathematical framework is inspired by the theory of model reduction by moment-matching and considers both continuous and discontinuous inputs, depicting an efficient and closed-form method to compute such a steady-state behaviour. This study approaches the design of an energy-maximising optimal controller for a single WEC device by employing the moment-based phasor transform, describing a pioneering application of this novel moment-matching mathematical scheme to an optimal control problem. Under this framework, the energy-maximising optimal control formulation is shown to be a strictly concave quadratic program, allowing the application of well-known efficient real-time algorithms.

1. Introduction

Energy capture from ocean waves has the potential to help fulfil the increasing worldwide energy demand, with an estimation of about 32.000 TWh/year (Mork, Barstow, Kabuth, & Pontes, 2010). Despite such a potential, wave energy is still at an early stage of development, since the technical and conceptual convergence to a technology best suited for this application has not yet been achieved (Edenhofer et al., 2011). Consequently, hundreds of patents, proposing different methodologies, have been filled all over the world (Pelc & Fujita, 2002). A noteworthy overview and classification of *Wave Energy Converters* (WECs) can be found in Falcão (2010).

In a more precise definition, a WEC is a device to harvest ocean wave energy by converting the mechanical energy of the waves to electrical energy by means of a *Power Take-Off* (PTO) system. In order to be profitable, an optimised process that ensures extracting the maximum time averaged power, for a given WEC device, from ocean waves is crucial. Moreover, in order to maximise power absorption and minimise the risk of damage, such an optimisation strategy must take into account the physical limitations of the whole conversion chain. Such an optimisation procedure can be achieved by designing an *optimal controller* that accomplishes such objectives.

A considerable number of optimal control formulations and methods have been studied and developed to maximise the energy extraction process from WECs, with extensive reviews available, for example in Ringwood, Bacelli, and Fusco (2014). One particular popular wave energy control strategy is Model Predictive Control (MPC). The success of MPC on the energy-maximising control is mainly due to its ability to handle physical constraints systematically and within a finite horizon optimisation process. While MPC applied to WECs also involves a mathematical model, a typical receding horizon strategy, and can deal with system constraints, the objective function contrasts significantly with the one related to the usual set-point tracking objective. Rather, a converted energy-maximising objective, consistent with the definition

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of the WEC control problem (see Section 3.3) is employed. In particular, this variation can cause numerical search problems, due to a potential loss of convexity of the performance function involved for this application (Faedo, Olaya, & Ringwood, 2017), compared to the normal quadratic form associated with tracking problems. In addition, the computational burden required for such a strategy can render the controller unsuitable for real-time applications (Faedo et al., 2017). Motivated by the appealing characteristics of MPC, several studies utilise "MPC-like" strategies, based on *spectral* and *pseudospectral* methods (Fahroo & Ross, 2008; Garg, Hager, & Rao, 2011), to try to overcome the (possibly) demanding computational effort of the original MPC optimal control formulation. A recent overview of both MPC and spectral and pseudospectral MPC-like strategies in wave energy applications can be found in Faedo et al. (2017). Notwithstanding, computing this energy-maximising control law in real-time is currently a strong concern among the wave energy community, and most of the proposed real-time strategies are usually inherently suboptimal.

Since the sea state (which directly affect the dynamic behaviour of WECs) varies slowly over time, the steady-state analysis of WECs is of paramount importance to design efficient real-time controllers for energy maximisation, as already exploited in studies such as Bacelli and Ringwood (2015) or Bacelli, Ringwood, and Gilloteaux (2011). Recently the *moment-based phasor transform* has been proposed to compute the steady-state response of a dynamical system under continuous or discontinuous inputs, see Scarciotti and Astolfi (2016b). From now on we refer to the framework induced by the moment-based phasor transform as the *moment-domain* characterisation (or formulation) of a system. This mathematical tool is based upon the theory developed in several studies concerning model order reduction (and particularly, moment-matching methods), such as Astolfi (2010) and Scarciotti and Astolfi (2015, 2016a).

In particular, in Scarciotti and Astolfi (2016b) it has been shown that the *phasors* of an electrical circuit are the *moments* computed at a single point on the imaginary axis of the transfer function of the linear system describing the circuit. Exploiting this relation, Scarciotti and Astolfi (2016b) has developed a mathematical framework to perform the steady-state analysis of systems driven by both continuous and discontinuous sources. The use of this framework is demonstrated in Scarciotti and Astolfi (2016b), both analytically and numerically, by analysing the steady-state behaviour of power inverters and wireless power transfer systems.

Nevertheless, and to the best of the authors knowledge, this moment-based framework has not yet been exploited to solve an optimal control problem. In this paper, we recognise the potential of such a mathematical tool to present a first application of the moment-based phasor transform for optimal control design, subject to path constraints. In particular, an energy-maximising optimal controller for a wave energy converter is designed, based on such a novel framework. Moreover, since the theoretical formulation is presented for a general class of devices, this paper not only demonstrates a single application case, but introduces the mathematical foundations for a novel approach to model-based optimal control design for WECs, in general.

The remainder of this study is organised as follows: first, basics of the moment representation of a system and its connection with the steady-state behaviour of a dynamical system are recalled in Section 2, while the WEC optimal control problem is described in Section 3. A novel moment-based approach for the solution of the optimal control problem for WECs is developed analytically in Section 4, constituting the main original contribution of the paper. Numerical examples of the application of the moment-based WEC control formulation, under different sea conditions, are given in Section 5, proving the efficacy of the approach, while conclusions on the overall application of the proposed method are provided in Section 6.

1.1. Notation and preliminaries

Standard notation is considered through this study, with some exceptions further detailed in this preliminary section. $\mathbb{R}^+$ ($\mathbb{R}^-$) denotes the set of non-negative (non-positive) real numbers. $\mathbb{C}^0$ denotes the set of pure-imaginary complex numbers and $\mathbb{C}^-$ denotes the set of complex numbers with negative real part. The symbol 0 stands for any zero element, according to the context. The symbol $\mathbb{I}_n$ denotes an order n identity matrix, while the notation $\mathbb{1}_{n\times m}$ is used to denote a $n \times m$ Hadamard identity matrix (i.e. a $n \times m$ matrix with all its entries equal to 1). The spectrum of a matrix $A \in \mathbb{R}^{n\times n}$, i.e. the set of its eigenvalues, is denoted as $\sigma(A)$. The symbol $\bigoplus$ denotes the direct sum of n matrices, i.e. $\bigoplus_{i=1}^{n} A_i = \mathrm{diag}(A_1, A_2, \dots, A_n)$. The notation $\Re\{z\}$ and $\Im\{z\}$, with $z \in \mathbb{C}$, stands for the ***real-part*** and the ***imaginary-part*** operators respectively, whilst $\mathscr{H}\{Z\} = \frac{Z+Z^\intercal}{2}$ stands for the ***symmetric-part*** of Z, where $Z \in \mathbb{R}^{n\times n}$. If $F \in \mathbb{R}^{n\times n}$ is a symmetric matrix, the expression $F \succ 0$ implies that F is positive-definite. The *Kronecker product* between two matrices $M_1 \in \mathbb{R}^{n\times m}$ and $M_2 \in \mathbb{R}^{p\times q}$ is denoted as $M_1 \otimes M_2 \in \mathbb{R}^{np\times mq}$. The convolution between two functions $f(t)$ and $g(t)$ over a finite range $[0, t]$, i.e. $\int_0^t f(\tau)g(t-\tau)d\tau$ is denoted as $f * g$. The inner product between two functions $w(t)$, $l(t) \in L^2(\mathbb{R})[a, b]$, where $L^2(\mathbb{R})[a, b]$ is the set of all real-valued functions square integrable in the interval $[a, b]$, is given by

$$\langle w(t), l(t)\rangle = \int_a^b w(\tau)l(\tau)\, d\tau. \tag{1}$$

If $\Omega : \mathscr{X} \longrightarrow \mathscr{Z}$ is a linear transformation, where $\mathscr{X}$ and $\mathscr{Z}$ are $\mathbb{K}$-vector spaces ($\mathbb{K}$ a field), the image and the kernel of Ω are denoted $\mathrm{Im}\{\Omega\} \subset \mathscr{Z}$ and $\mathrm{Ker}\{\Omega\} \subset \mathscr{X}$, respectively. Finally, the symbol $\varepsilon_n \in \mathbb{R}^{n\times 1}$ denotes a vector with odd components equal to 1 and even components equal to 0.

In the remainder of this section the formal definitions of two important operators are presented, since their definition in the literature can be often ambiguous.

Definition 1 (*Brewer, 1978 Kronecker Sum*). The *Kronecker sum* between two matrices P_1 and P_2, with $P_1 \in \mathbb{R}^{n\times n}$ and $P_2 \in \mathbb{R}^{k\times k}$, is defined (and denoted) as

$$P_1 \hat{\oplus} P_2 \triangleq P_1 \otimes \mathbb{I}_k + \mathbb{I}_n \otimes P_2. \tag{2}$$

Definition 2 (*Brewer, 1978 Vec Operator*). Given a matrix $H = [h_1, h_2, \dots, h_n] \in \mathbb{R}^{n\times m}$, where $h_j \in \mathbb{R}^n$, $j = 1, \dots, m$, the vector valued operator *vec* is defined as

$$\mathrm{vec}\{H\} \triangleq \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix} \in \mathbb{R}^{nm}. \tag{3}$$

Finally, useful theorems and properties of the Kronecker sum, and the vec and Hermitian-part operators, are recalled in the following.

Theorem 1 (*Brewer, 1978*). *Consider matrices P_1 and P_2 as in Definition 1. Assume that P_1 and P_2 have eigenvalues λ_i, for $i = 1, \dots, n$, and μ_j, for $j = 1, \dots, k$. Then the Kronecker sum $P_1 \hat{\oplus} P_2$ has the nk eigenvalues*

$$\lambda_1 + \mu_1, \dots, \lambda_1 + \mu_k, \lambda_2 + \mu_1, \dots, \lambda_2 + \mu_k, \dots, \lambda_n + \mu_k. \tag{4}$$

Corollary 1 (*Brewer, 1978*). *The Kronecker sum $P_1 \hat{\oplus} P_2$ is invertible if and only if $\sigma(P_1) \cap \sigma(-P_2) = \emptyset$.*

Property 1 (*Brewer, 1978*). *Let $A \in \mathbb{R}^{n\times m}$ and $B \in \mathbb{R}^{p\times q}$. The following relation for the vec operator holds:*

$$vec\{AB\} = (\mathbb{I}_q \otimes A)vec\{B\} = (B^\intercal \otimes \mathbb{I}_n)vec\{A\}. \tag{5}$$