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PII: S0022-5096(18)30573-8
DOI: https://doi.org/10.1016/j.jmps.2018.09.006
Reference: MPS 3437

To appear in: *Journal of the Mechanics and Physics of Solids*

Received date: 2 July 2018
Revised date: 28 August 2018
Accepted date: 5 September 2018

Please cite this article as: Paul Greaney, Martin Meere, Giuseppe Zurlo, The out-of-plane behaviour of dielectric membranes: description of wrinkling and pull-in instabilities, *Journal of the Mechanics and Physics of Solids* (2018), doi: https://doi.org/10.1016/j.jmps.2018.09.006

# The out-of-plane behaviour of dielectric membranes: description of wrinkling and pull-in instabilities

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**Abstract**

Voltage controlled dielectric membranes exhibit two fundamental types of instability, strongly affecting their performances: the occurrence of wrinkling, which is due to membranal compressive stresses, and the onset of pull-in, a catastrophic thinning localisation that preludes electrical breakdown. In this manuscript we provide a unifying energetic description of both instabilities for large, out-of-plane and inhomogeneous deformations. By using the ideas of relaxation and regularisation of the energy, originally proposed by Pipkin (IMA J. Appl. Math. 36, 85-99, 1986) and Hilgers and Pipkin (Quart. Appl. Math. 50, 389-400, 1992) for purely elastic membranes, we show that the onset and development of wrinkling can be effectively described by the relaxed electroelastic energy. For axially symmetric membranes and neo-Hookean materials, we show that pull-in corresponds to failure of the strong ellipticity condition of the regularised electroelastic energy, thus extending to out-of-plane deformations the validity of a previous estimate for planar systems (Phys. Rev. Lett., 118(7) 078001, 2017). In agreement with ubiquitous experimental evidence, we also show that wrinkled states are always stable below the pull-in voltage. Our theoretical findings are assessed by the comparison with experiments on out-of-plane, voltage-actuated annular membranes, showing good agreement both in terms of description of wrinkled states, and for the prediction of the pull-in instability.

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