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Equilibrium selection, observability and backward-stable solutions[☆,☆☆]

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ABSTRACT

The robustness of stability under learning to observability of exogenous shocks is examined. Regardless of observability assumptions, the minimal state variable solution is robustly stable under learning provided the expectational feedback is not both positive and large, while the nonfundamental solution is never robustly stable. Overlapping generations and New Keynesian models are considered and concerns raised in Cochrane (2011, 2017) are addressed.

1. Introduction

The connection between shock observability and equilibrium selection under learning is explored in this paper. Under rational expectations (RE) shock observability does not affect equilibrium outcomes if current aggregates are observable, but stability under adaptive learning is altered. The common assumption that exogenous shocks are observable to forward-looking agents has been questioned by prominent authors, including Cochrane (2009) and Levine et al. (2012). We examine the implications for equilibrium selection of taking the shocks as unobservable.

It is useful first to recall some standard terminology. An equilibrium, viewed here as a stochastic process, is called "explosive" if it is unbounded (almost surely) as time goes to infinity; otherwise it is called "non-explosive." In Section 2 we also refer to non-explosive solutions as "forward-stable" and we define "backward-stable" in an analogous fashion. A linear model is said to be "determinate" if it has a unique non-explosive equilibrium; it is called "indeterminate" if it has multiple non-explosive equilibria; and it is called "explosive" otherwise.

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With this terminology our results are easily summarized. Using a simple model, we establish that determinacy implies the existence of a unique rational expectations equilibrium that is robustly stable under learning. Our results also cover the indeterminate case. We then examine the concerns raised in Cochrane (2009, 2011, 2017) about the new-Keynesian model. These concerns arise in large part by his adoption of RE as a modeling primitive. We view RE as more naturally arising as an emergent outcome of an adaptive learning process, and we find that by modeling agents as adaptive learners Cochrane's concerns vanish.

To elaborate more fully, the frequent presence of multiple equilibria in rational expectations models has resulted in debates on selection criteria. In the determinate case it is common practice to pick the unique non-explosive RE solution. This choice has theoretical support when the explosive paths can be shown not to be legitimate equilibrium paths based on transversality conditions or on no-Ponzi-game or other constraints. However, there are cases in which explosive RE paths meet all the relevant equilibrium conditions. Furthermore, there are many models that are indeterminate, i.e. have multiple non-explosive RE solutions.

Adaptive learning provides a natural equilibrium selection device. Under adaptive learning agents are assumed to form expectations using forecasting models, which they update over time in response to observed data. There is a well-developed theory that allows the researcher to assess whether agents, using least-squares updating of the coefficients of their forecasting model, will come to behave in a manner that is asymptotically consistent with RE, i.e. whether the rational expectations equilibrium (REE) is stable under learning: see Marcet and Sargent (1989) and Evans and Honkapohja (2001).[1] For the cases of equilibrium multiplicity studied in this paper we adopt stability under adaptive learning as our selection criterion.

Both RE and the adaptive learning approach require precise specification of the information structure. A common assumption is that agents are able to condition time t forecasts on t-dated exogenous shocks. While in some environments this assumption may be quite natural, in others, e.g. for aggregate productivity or taste shocks, it may be difficult to defend. These issues have been raised recently in policy-related literature: Cochrane (2009, 2011) has argued that the New Keynesian model's exogenous shocks, specifically the monetary policy shocks, are most naturally taken as unobservable. RE models that exclude from information sets certain contemporaneous variables have been prominent: Lucas et al. (1973) did precisely this in his famous islands model, and Mankiw and Reis (2002) have exploited the same idea in their sticky information DSGE environment. A central part of this paper is to explore the connection between observability and equilibrium selection.

In Section 2 we develop a general theory within the context of a simple forward-looking model with an exogenous shock. We identify two REE of interest: the fundamental, or "minimal state variable" (MSV) solution, which is always non-explosive; and a "non-fundamental" (NF) solution, which may or may not be non-explosive. We then turn to stability under learning. In a model with observable shocks, McCallum (2009a) showed that determinacy implies that only the MSV solution is stable under learning. However, Cochrane (2009, 2011) argued that McCallum's stability results hinged on observability of these shocks. We study stability under learning of the two solutions, taking into account the possibility of unobserved exogenous shocks. The central result of our paper is that, regardless of the observability assumption, the MSV solution is robustly stable under learning, provided only that the positive feedback from expectations is not too large. In contrast the NF solution is never robustly stable under learning.

While our analysis is done in a general framework, the motivation for this study is the collection of issues raised by Cochrane (2009, 2011, 2017) in connection with the NK model. To address these issues, Section 3 studies three applications. The first is the flexible-price NK model employed by McCallum (2009a, 2009b) and Cochrane (2009) in their interchange.[2] We find that provided the interest-rate rule satisfies the Taylor principle the MSV solution is both the unique non-explosive solution and the unique robustly stable REE under learning, regardless of shock observability. In particular, in the NK model adaptive learning selects the REE solution typically adopted by practitioners. The second application is a discrete-time version of the sticky-price NK model used by Cochrane (2017) to study the backward-stable solution under an interest-rate peg. We demonstrate that this solution is precisely the MSV solution and that it is not stable under learning. The last application is an overlapping generations model with production and money. This model can be either determinate (in which case the MSV solution is the unique non-explosive solution) or indeterminate (in which case the NF solution is also non-explosive). We show that in either case the MSV solution is robustly stable under learning and the NF solution is not.

2. Model and results

Our key results can be presented most effectively in a linear univariate framework. The model is given by

$$y_t = \beta E_t y_{t+1} + v_t \text{ and } v_t = \rho v_{t-1} + \varepsilon_t, \text{ where } 0 < \rho < 1. \tag{1}$$

Here v_t captures a positively autocorrelated stationary exogenous process, with ε_t zero mean, *iid* and having bounded support. The parameter β measures the expectational feedback in the model. We exclude the non-generic cases in which $|\beta| = 1$ or $\beta\rho = 1$. The expectational feedback parameter β plays an important role in the assessment of equilibrium multiplicity.

[1] Using the standard linear three-equation reduced-form set-up, stability under adaptive learning in New Keynesian models was first studied by Bullard and Mitra (2002).

[2] McCallum (2012) provides detailed comments on this interchange and discusses further the issue of equilibrium selection.

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