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# Revisiting operating cost in resource extraction industries

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ABSTRACT

A modified version of the Cobb-Douglas production function is proposed for simulating production costs in resource extraction models. The resulting average cost function is U-shaped with a wide bottom, and as such should be more representative of the economies of scale associated with bulk operations. It also possesses a minimum which is obtainable from the characteristics of the operations. The viability of the proposed cost function is demonstrated in a profit maximisation exercise constructed as a problem in optimal control, rendering results consistent with what could be seen in a real-world resource extraction operation with similar constraints.

## 1. Introduction

A resource extraction model provides the backbone for the supply-side of a resource trade model, which is often employed in the study of commodity markets. The mine operator must cover the costs of delivering minerals to the mine gate where the product is sold to an offtake agent at the prevailing market price. In modelling applications it is often desirable to have a cost function explicitly dependent on product output, such as in the coal market study of Haftendorn et al. (2010) and other production scheduling optimisation models where output represents a choice variable. Operating cost is of interest for other factors governing resource supply, such as the cut-off grade parameter in metals mining which determines the level of mineralisation required in order for extraction to proceed, see Zhang and Kleit (2016). Operating cost is also a determinant of the optimal price threshold for mine activation in real option value models, such as used in Zhang et al. (2014) as well as in Zhang et al. (2017).

The average cost derived from a total production cost curve is thought to be U-shaped and can be represented by a quadratic function. Though computationally convenient to work with, one is then faced with the problem of determining the turning point of the average cost function. Also, it is questionable whether a quadratic average cost renders a wide enough U-shape such that it is adequate in representing costs in bulk mining operations.

We propose a modified version of the Cobb-Douglas production cost function, in order to better approximate the economies of scale associated with resource extraction operations for applications requiring a cost function explicitly dependent on production quantity. The generalised Cobb-Douglas production function is used here due to its relative simplicity, good behaviour and the separability of input factors. A limiting term is introduced to account for diseconomies of scale that emerge as production levels approach technical capacity.

## 2. Relevant literature

The appropriate functional form for industrial production cost curves has been under debate since Cobb and Douglas (1928) first proposed what has since become a widely used means for characterising the production behaviour of the firm. They proposed relating production $Q$ to quantities of inputs, specifically labour $L$ and capital $K$, via the equation

$$Q = AL^{\alpha}K^{\beta}, \tag{1}$$

where $A > 0$ indicates the state of technology, and $\alpha, \beta > 0$ denote the relative factor shares in the case of perfect competition. Eq. (1) is a homogeneous function of degree $\alpha + \beta$; returns to scale are constant when $\alpha + \beta = 1$, increasing when $\alpha + \beta > 0$, and decreasing when $\alpha + \beta < 0$. In addition, Eq. (1) is additively separable and homothetic (Katzner, 1970). The constant-elasticity-of-substitution (CES) production function (Arrow et al., 1961) generalises the Cobb-Douglas. The merits of using the Cobb-Douglas production function for analyses of production processes is argued by Bhanumurthy (2002).

The generalised Cobb-Douglas production function was studied by Vîlcu (2011) in a differential geometry setting, rendering interesting links, and in particular, that this function exhibits “constant returns to scale if and only if the corresponding hypersurface is developable”

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Vîlcu (2011).

The historical development of the production function is discussed in Mishra (2007), the relevance of technical and allocative efficiencies is highlighted, and the production function is more precisely defined as relating production inputs to the maximal technically feasible level of output.

Capacity-constrained production cost is typically considered by means of constraints imposed on an optimisation problem setting, as in Florian and Klein (1971), and Lambrecht and Vander Eecken (1977), and do not feature explicitly in the cost function itself.

The cost and production data typically available for modelling purposes are in many cases inadequate for specifying certain cost functional forms. As such, researchers have incorporated simpler cost functions in their models of commodity production and trade. Constant per-unit cost values have been used in, for instance, Leuthold et al. (2008), Holz et al. (2008), and Aune et al. (2004). Linear cost functions have been incorporated to account for changes in cost resulting from varying production levels, as in Yang et al. (2002) and Haftendorn et al. (2010). A quadratic marginal cost ($MC$) formulation has also been suggested, see Poulizac et al. (2012). However, for a quadratic $MC$ one is faced with the problem of establishing the turning point of the $ATC$ function. Golombek et al. (1995) propose a form for the marginal production cost where the cost increases exponentially as the quantity produced approaches the firm's maximum production capacity limit. This approach is adopted by Paulus and Trüby (2011).

## 3. Producer behaviour theory

Definitions and explanations of applicable microeconomic theory can be sourced from texts such as Bernheim and Whinston (2008), or Goolsbee et al. (2013).

A producer's operating cost of supply as a function of quantity produced $q$ is a *total cost*, $TC(q)$, comprising *variable cost*, $VC(q)$, and *fixed cost*, $FC$. Variable costs are those costs that vary with the quantity of output produced, such as fuel, labour and consumables. Fixed costs on the other hand are expenses incurred regardless of the quantities produced and include overhead items such as the cost of renting commercial space or the cost of assets such as premises or capital equipment. This distinction is important in the context of loss-making producers and determining when these producers would begin cutting back on output. In the short-run,[1] loss-making suppliers would cut back volumes only when the average loss on those volumes exceeds the average variable cost, $AVC = VC/q$, associated with producing those volumes. Loss-making firms are able to increase output to spread $FC$ over volumes, thereby reducing $AFC$. In the case where the firm has a higher $FC$, the firm might be inclined to maintain loss-making production volumes for longer. In the long-run, a firm would exit the industry if the market price $P$ is below its long-run $ATC$.

A producer's *marginal cost* of production is simply the derivative of his total cost function, i.e.

$$MC = \frac{\partial TC}{\partial q}.$$

$ATC$ is typically assumed to exhibit a U-shape.

## 4. Specifying cost function form

The chosen form for operating cost function (3) is motivated by consideration of the Cobb-Douglas production function, which is a particular case of the CES or *constant elasticity of substitution* functions. The general CES function is given by (Ferris and Pang, 1995):

$$\left(\sum_{i=1}^{I} \lambda_i x_i^{p}\right)^{\frac{1}{p}} \quad \lambda_i \geq 0, \quad i = 0, \ldots, I$$

assumed to be defined over $\mathbb{R}_+^I$, with continuity-defined boundary values. CES functions are concave for $p \leq 1$ and convex for $p \geq 1$. Then the Cobb-Douglas function is defined as (Ferris and Pang, 1995):

$$q = \prod_{i=1}^{I} x_i^{\phi_i}, \tag{2}$$

where $q$ is the total quantity of product produced, $x_i$ is the $i^{th}$ input item, and

$$\phi_j \equiv \frac{\lambda_j}{\sum_{i=1}^{I} \lambda_i}.$$

We implement a modified version of the Cobb-Douglas by adding a term $\frac{a}{Q-q}$, where $a$ is some positive constant, to account for cost increases owing to congestion as production levels $q$ approach the maximum physical capacity output $Q$ in the short-run. This function exhibits a U-shape with a longer, flatter bottom than the quadratic form and should as such be more representative of the economies of scale typical of bulk resource extraction operations.

That is, the miner is able to reduce per-unit costs by spreading overall costs over larger volumes, facing a limit to which production volumes can increase before inefficiencies begin to arise. The proposed production cost function is of the form:

$$TC = a \cdot q^b + k + \frac{\epsilon \cdot q}{Q - q}, \tag{3}$$

so that

$$AC = a \cdot q^{b-1} + \frac{k}{q} + \frac{\epsilon}{Q - q} \tag{4}$$

and

$$MC = ab \cdot q^{b-1} + \frac{\epsilon Q}{(Q - q)^2}. \tag{5}$$

These functions are plotted in Fig. 1. Technical efficiency is assumed so that the analysis of the production function is concerned with allocative efficiency solely (Mishra, 2007).

### 4.1. Motivation for proposed cost functional form

Cost function (3) can be derived from the Cobb–Douglas production function by re-writing (2) as

$$q = \mu x_1^{\alpha_1} \ldots x_n^{\alpha_n},$$

with $\mu$ some positive constant, and $x_i \in X$, where $X$ is a vector space of dimension $n+1$. For $\sum_i \alpha_i = 1$, returns-to-scale are constant, for $\sum_i \alpha_i > 1$, returns are increasing, and for $\sum_i \alpha_i < 1$ returns are decreasing. Now let $\frac{1}{\mu} q = \tilde{q}$, then

$$\tilde{q}^{\beta_1} \ldots \tilde{q}^{\beta_n} = x_1^{\alpha_1} \ldots x_n^{\alpha_n},$$

with $\sum_{i=1}^{n} \beta_i = 1$. Linearising the above equation as

$$\ln \tilde{q} = \beta_1 \ln \tilde{q} + \ldots + \beta_n \ln \tilde{q} = \alpha_1 \ln x_1 + \ldots + \alpha_n \ln x_n,$$

and letting $\ln \tilde{q} = Q$, then

$$\bar{\beta} \cdot \bar{Q} = \bar{\alpha} \cdot \bar{x} \quad \text{where } \bar{Q} = Q(1, \ldots, 1).$$

That is, the problem is then one of finding basis $\{\beta_1, \ldots, \beta_n\}$ in $X$ such that the hyperplane $\bar{\beta} \cdot \bar{Q}$ coincides with that of $\bar{\alpha} \cdot \bar{x}$. This is achieved by choosing

$$\beta_i = \frac{\alpha_i \ln x_i}{\ln \tilde{q}},$$

so that

[1] The short-run is the nearer term during which a firm is unable to make adjustments to its capital of production, whereas in the long-term the firm has sufficient time to invest and adjust its means of production.