Annals of Nuclear Energy 123 (2019) 145–152

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Pin-by-pin power reconstruction method using expansion in pseudo-harmonics

Danielle G. Teixeira *, Fernando C. da Silva

Nuclear Engineering Program– COPPE, Universidade Federal do Rio de Janeiro, Av. Horácio Macedo, 2030, 21941-972 Rio de Janeiro, RJ, Brazil

ARTICLE INFO

Article history:
Received 4 July 2018
Received in revised form 27 August 2018
Accepted 8 September 2018

Keywords:
Pseudo-harmonics method
Pin power reconstruction
Nodal expansion method
Neutron diffusion theory
Nuclear power density

ABSTRACT

A pin-by-pin power reconstruction method was developed using the expansion in eigenfunctions for the reconstruction of neutron flux distribution in a homogeneous fuel assembly. The eigenfunctions used were the so-called pseudo-harmonics, which are eigenfunctions associated with the leakage plus removal operator of the neutron diffusion theory. The modulation technique was used to obtain the heterogeneous distribution of power density. In this technique, the power density homogeneous distribution, calculated with the reconstructed neutron flux is multiplied by a form function. This form function is obtained in the calculation of assembly homogenization. The results obtained by this method show good agreement between the reconstructed pin-by-pin power and that of the reference heterogeneous calculation, for the benchmark used. The largest deviation found was of the order of 4.7%, in a peripheral cell of a fuel element with the faces towards the region of the baffle/reflector.

1. Introduction

Modern nodal methods, such as the Nodal Expansion Method (NEM) (Finnemann et al., 1977) are quite fast and very accurate, but provide only average flux values (in node volume and surfaces), in addition to the effective multiplication factor.

However, for safety analysis (Todreas and Kazimi, 2001) it is necessary to know the power distribution in each fuel assembly (at least the average in each fuel pin cell) Therefore, it is necessary to use a pin-by-pin power reconstruction method, after nodal calculations.

Over the last 40 years several pin-by-pin power reconstruction methods have been developed (Koebke and Wagner, 1977; Koebke and Hetzel, 1985; Rempe et al., 1988; Böer and Finnemann, 1992; Joo et al., 1999; Jung and Cho, 1991; Joo et al., 2009; Yu et al., 2010, 2010,; Pessoa et al., 2015; Pessoa et al., 2016). These methods use from polynomial expansions to analytical solutions of the diffusion equation to obtain neutron flux distribution of a homogeneous fuel assembly. Some methods such as, for instance, those of the references (Koebke and Wagner, 1977; Koebke and Hetzel, 1985; Rempe et al., 1988; Böer and Finnemann, 1992; Joo et al., 1999; Joo et al., 2009; Yu et al., 2010, 2010,; Pessoa et al., 2015) make use of the modulation technique (Koebke and Wagner, 1977) and, consequently, use nuclear parameters from the assembly homogenization process (Koebke, 1978; Smith, 1986), from which the form functions are also derived. On the other hand, the methods that do not use the modulation technique, use nuclear data of the homogenized pin cell (Jung and Cho, 1991; Pessoa et al., 2016).

The reconstruction method developed, and presented in this paper, uses nuclear parameters of the homogenized fuel assembly and, consequently, the modulation technique. In addition, the results of the reactor global calculation, carried out with the nodal method NEM with either, the average flux in the node volume, the average partial currents on the node surfaces or the effective multiplication factor, are also used. It is noteworthy that the nodal calculations are carried out for nodes with the same dimensions of the fuel assembly's base area and heights of 30–40 cm.

The pseudo-harmonics (Gomit et al., 1985; Silva et al., 1987; Abreu et al., 1989; Abreu et al., 1990) are calculated for each node, after integration along its height, i.e., only in (x,y). The pseudo-harmonics are the eigenfunctions used in the expansion for the calculation of neutron flux distribution in the node (homogeneous fuel assembly). As the pseudo-harmonics are eigenfunctions associated with the leakage plus removal term of the neutron diffusion theory, the equation of two-dimensional neutron diffusion is used in the calculation of the expansion coefficients. It must be highlighted that this procedure is carried out for each node in which the fuel assembly was divided in the axial direction.

It should be noted that the pseudo-harmonic method was originally conceived by Gomit et al. (1985), however, since the second

* Corresponding author.
E-mail addresses: dteixeira@nuclear.ufrj.br (D.G. Teixeira), fernando@nuclear.ufrj.br (F.C. da Silva).

https://doi.org/10.1016/j.anucene.2018.09.010
0306-4549/

half of the 1980s, it has been modified to improve performance, as can be seen in Silva et al. (1987), Abreu et al. (1989) and Abreu et al. (1990) and cited by Planchard (1995), which highlights the advancement of the method originally developed by him. Thus, the reconstruction method presented in this article uses the pseudo-harmonics of this improved calculation (Silva et al., 1987; Abreu et al., 1989).

In addition, the method proposed in this paper uses an analytical solution of the two-dimensional diffusion equation only to generate flux distribution on the surfaces of the homogeneous fuel assemblies. The surface net currents of each node as well as the homogeneous fluxes in the corners of those nodes were also used in the calculations as boundary conditions to obtain the analytical solution. The flux in each node corner is an essential information for some reconstruction methods (Rempe et al., 1988; Böer and Finnemann, 1992; Pessoa et al., 2015). In the proposed method, flux calculation in the corners uses the method of Rempe et al. (1988) modified.

Therefore, the pin-by-pin heterogeneous power density reconstruction method, using the expansion in pseudo-harmonics will be called, from now on, PHRM. The innovative proposal of the PHRM is to generate homogeneous flux distribution inside each homogeneous fuel assembly using an expansion in pseudo-harmonics in combination with the analytical solution of the diffusion equation and the modified method by Rempe et al. (1988), for the calculation of the flux in the node corners.

This paper is organized as follows: Section 2 presents the approach of neutron diffusion equation discretization, for the calculation of pseudo-harmonics. How the coefficients of expansion in pseudo-harmonics are determined is the subject of Section 3. The determination of flux distribution on the surfaces of the homogeneous fuel assembly is presented in Section 4. On the other hand, the distribution of the reconstructed pin-by-pin is presented in Section 5. The details of the benchmark used as well as the results of the method developed are presented and discussed in Section 6. Final considerations and conclusions are given in Section 7.

2. Homogeneous flux expansion in Pseudo-Harmonics

Neutron flux distribution, for two energy groups, in each node of a fuel assembly, after integration along the height of the node, is the solution of the following equation:

$$-D_g\frac{\partial^2}{\partial x^2}\phi_{g,\hom}(x,y)-D_g\frac{\partial^2}{\partial y^2}\phi_{g,\hom}(x,y)+\Sigma_{Rg}\phi_{g,\hom}(x,y)$$
$$=\frac{1}{k_{eff}}\chi_g\sum_{g'=1}^{2}\nu\Sigma_{fg'}\phi_{g',\hom}(x,y)+\sum_{\substack{g'=1\\g'\neq g}}^{2}\Sigma_s^{g'\to g}\phi_{g',\hom}(x,y) \quad (1)$$

where:

$$\Sigma_{Rg}=\Sigma_{ag}+\sum_{\substack{g'=1\\g'\neq g}}^{2}\Sigma_s^{g\to g'}+D_gB_z^2, \quad (2)$$

with B_z^2 given by

$$B_z^2\equiv\frac{\bar{J}_{gzr}-\bar{J}_{gz\ell}}{a_z\bar{\phi}_g}, \quad (3)$$

where a_z is the height of the node and $\bar{\phi}_g$ the average flux in the node volume, while $\bar{J}_{gz\ell}$ and $\bar{J}_{gzr}$ are the average net currents in the node lower and upper surfaces, respectively. The magnitudes k_{eff} (present in Eq. (1)), $\bar{\phi}_g$ and $\bar{J}_{gus}$; for $u=x,y,z$ and $s=\ell,r$, are derived from the nodal calculation using the NEM method. In addition, the nuclear parameters (uniform) that appear in Eqs. (1) and (2) are the same used in coarse-mesh nodal calculation.

For the calculation of the homogeneous flux distribution, which will be used in pin-by-pin power reconstruction, Eq. (1) is discretized by finite differences, with scheme centered on the mesh (Nakamura, 1977). Thus, for a N × N mesh, where N^2 is the number of pin cells in the fuel assembly, the following system of equations is obtained:

$$\begin{bmatrix}B_1 & 0\\ 0 & B_2\end{bmatrix}\begin{bmatrix}\underline{\phi}_1\\ \underline{\phi}_2\end{bmatrix}=\frac{1}{k_{eff}}\begin{bmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{bmatrix}\begin{bmatrix}\underline{\phi}_1\\ \underline{\phi}_2\end{bmatrix}+\begin{bmatrix}0 & S_{12}\\ S_{21} & 0\end{bmatrix}\begin{bmatrix}\underline{\phi}_1\\ \underline{\phi}_2\end{bmatrix}+\underline{s}, \quad (4)$$

where the matrices B_g, $F_{gg'}$ and $S_{gg'}$ represent the discretization of the leakage plus removal, fission and scattering terms, respectively. While

$$\underline{s}\equiv\frac{2}{A_{cel}}\begin{bmatrix}D_1(L_{y\ell}\underline{\psi}_{1y\ell}+L_{x\ell}\underline{\psi}_{1x\ell}+L_{xr}\underline{\psi}_{1xr}+L_{yr}\underline{\psi}_{1yr})\\ D_2(L_{y\ell}\underline{\psi}_{2y\ell}+L_{x\ell}\underline{\psi}_{2x\ell}+L_{xr}\underline{\psi}_{2xr}+L_{yr}\underline{\psi}_{2yr})\end{bmatrix}, \quad (5)$$

where A_{cel} is the area of the pin cell. Vector $\underline{\phi}_g$ is a column matrix with N^2 rows, of the following form:

$$\underline{\phi}_g\equiv\begin{bmatrix}\bar{\phi}_{g,\hom}^{1,1} & \bar{\phi}_{g,\hom}^{1,2} & \bar{\phi}_{g,\hom}^{1,3} & \cdots & \bar{\phi}_{g,\hom}^{i,j} & \cdots & \bar{\phi}_{g,\hom}^{N,N}\end{bmatrix}^T, \quad (6)$$

where $\bar{\phi}_{g,hom}^{i,j}$ represents the average flux in the pin cell (i,j). On the other hand $\underline{\psi}_{gus}$; for $u=x,y$ and $s=\ell,r$, is a column matrix with N rows whose components are shown in Fig. 1. Matrices L_{us} are filled with zeros, except for elements (i,j), which are equal to 1, where $\psi_{gus}^{i,j}$ contributes to $\bar{\phi}_{g,hom}^{i,j}$.

The solution of Eq. (4) is obtained using an expansion in pseudo-harmonics, of the following form:

$$\begin{bmatrix}\underline{\phi}_1\\ \underline{\phi}_2\end{bmatrix}=\sum_{n=1}^{M}\begin{bmatrix}c_{1,n}\,\underline{\Gamma}_{1,n}\\ c_{2,n}\,\underline{\Gamma}_{2,n}\end{bmatrix}, \quad (7)$$

where $M=N^2$ and $\underline{\Gamma}_{g,n}$, the pseudo-harmonics (column matrices with M rows), are solutions of the following eigenvalue problem:

$$B_g\underline{\Gamma}_{g,n}=\lambda_{g,n}\underline{\Gamma}_{g,n};\ g=1,2\ \text{e}\ n=1,M. \quad (8)$$

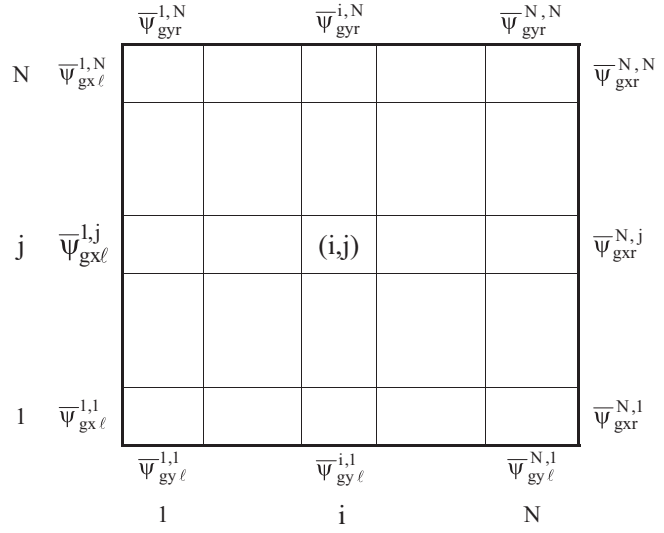

Fig. 1. Homogeneous FA discretized in meshes identical to pin cells.