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Original contribution

Compressive sensing image recovery using dictionary learning and shape-adaptive DCT thresholding

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ABSTRACT

Compressed sensing (CS) has shown to be a successful technique for image recovery. Designing an effective regularization term reflecting the image sparse prior information plays a critical role in this field. Dictionary learning (DL) strategy alleviates the drawback of fixed bases. But the structure information of the image is easy to be blurred in complex regions due to the absence of sparsity in dictionary learning. This paper proposes a novel joint dictionary learning and Shape-Adaptive DCT (SADCT) thresholding method. We first propose to exploit sparsity of image in shape-adaptive regions, which is beneficial to medical images of complex textures. In this framework, the local sparsity depicts the smoothness redundancies exploited by dictionary learning. Moreover, the sparsity is enhanced especially in detail areas by the newly introduced SADCT thresholding. The attenuated SADCT coefficients are used to reconstruct a local estimation of the signal within the adaptive-shape support. Image is represented sparser in SADCT transform domain and the details of the image information can be kept with a much larger probability. Based on split Bregman iterations, an efficient alternating minimization algorithm is developed to solve the proposed CS medical image recovery problem. The results of various experiments on MR images consistently demonstrate that the proposed algorithm efficiently recovers MR images and shows advantages over the current leading CS reconstruction approaches.

1. Introduction

Compressed sensing (CS) [1,2] reconstructs the signal perfectly through the non-linear reconstruction algorithm by using the sparse characteristics of the signal. As one of the most important applications, reconstructing image from incomplete measurements has always been a topic of great interest due to its significance in various applications. For example, it is well- known that MRI scans can cause physical damage to the patient, and the application of CS means that we can use much less radiation than traditional methods of data collection [3,4]. Since exploiting the prior knowledge of the original signal plays a substantial role in CS theory, it is necessary to utilize some constraints or prior knowledge to make up for the missing information. Various efforts have been made to look for an effective sparse constraint or realistic model. With such a sparse constraint or realistic model, CS reconstruction is formulated as a constrained optimization problem, which can be solved by various methods, such as iterative shrinkage/thresholding (IST) methods [5–7] and Bregman iterative algorithms [8,9].

Finding an optimal sparse representation of image is an active research area in compressed sensing MRI (CSMRI), since a sparser representation usually leads to lower reconstruction error [10,11]. There are many prespecified sparsifying dictionaries (basis or frame), e.g., Fourier transform, discrete cosine transform, wavelets, ridgelets [12], curvelets [13], contourlets [14] and shearlets. In spite of being simple and fast computation, the analytically designed dictionaries usually capture only one type of image features and lack adaptivity to the image local structures. Due to above shortcomings, reconstruction qualities of those dictionaries are not satisfactory. Adaptive dictionary learning [10,15–17] with the capability of better matching the contents of the images can sparsify image better since they are learnt for the particular image instances or class of images. The K-SVD [18] is a typical dictionary learning method which has been applied in CSMRI for a single image [10,19–21] and has significantly improved the reconstructed image quality than those using prespecified dictionaries [22,23]. However, the structure information of the image is easy to be blurred based on the dictionary learning method of overlapping square patches. Especially with the presence of singularities or edges, ringing artifacts arising from the Gibbs phenomenon become visible because of the lack of sparsity.

In the last decade, some research works have been made towards the development of shape-adaptive transforms. The core is to construct a framework that can be used for the analysis of arbitrarily shaped

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image segments, where the data show some unified forms. An attractive approach, named Shape-Adaptive DCT (SADCT), is proposed by Sikora et al. [24,25]. SADCT is computed by cascaded application of one-dimensional varying-length DCT transforms first on the columns and then on the rows that constitute the considered region. Actually, the use of a transform with a shape-adaptive support involves two independent problems: shape-adaptive transform and image-adaptive shape. In this paper, SADCT transform is used to solve the first problem and the Anisotropic Local Polynomial Approximation (LPA) -Intersection of Confidence Intervals (ICI) [26–28] is used to solve the second problem. SADCT has received considerable interest from the MPEG community, eventually becoming part of the MPEG-4 standard [29,30]. Furthermore, because of the better decorrelation and energy compaction properties, SADCT has shown to provide an effective method for image denoising [31]. Although SADCT features a remarkable potential for video compression applications and gets the initial realization for image denoising, this potential has been apparently ignored in the image recovery community.

In this paper, we integrate the dictionary learning (DL) and SADCT technique into a unified framework to reconstruct MR image from highly undersampled data. The dictionary trained by K-SVD characterizes the local sparsity that depicts the smoothness redundancies. Meanwhile, LPA-ICI finds the adaptive-shape region in the image patch, and then SADCT is performed in the found region to further exploit and enhance image sparsity. The proposed method has two benefits. First, because of the better decorrelation and energy compaction properties in the adaptive support set, which is shown in some unified forms. In this way, the sparsity of complex regions is enhanced by the newly introduced SADCT. As a new regularization term, SADCT can alleviate the blurred structure information of the image resulting from the dictionary learning method of overlapping square patches. Second, the proposed CS image recovery problem is formulated in the form of minimization functional under regularization-based framework. Based on split Bregman iterations, an efficient alternating minimization algorithm is developed to solve the above underdetermined inverse problem, which can achieve a fast and stable solution. By combining dictionary learning and SADCT, the data are represented sparser than dictionary learning itself. Due to the lack of sparsity in singularities or edges, the original ringing artifacts arising from the Gibbs phenomenon caused by dictionary learning can be alleviated.

The rest of this paper is organized as follows. Section 2 reviews the previous work in CSMRI (compressed sensing MRI) and states dictionary learning and Shape-Adaptive DCT. The proposed model jointing dictionary learning and Shape-Adaptive DCT thresholding method for image recovery, which applies the split Bregman iterations method, is detailed in Section 3. Section 4 demonstrates the performance of the proposed algorithm on various experiments under a variety of sampling schemes and undersampling ratios. Conclusions and future work are presented in Section 5.

2. Background

In this section, we review the previous classical work in CSMRI (compressed sensing MRI) and state dictionary learning and SADCT. The P-pixel 2D image to be reconstructed is denoted by $x \in \mathbb{C}^P$, and $f \in \mathbb{C}^m$ represents the undersampled Fourier measurements. x and f are related as $F_u x = f$, where $F_u \in \mathbb{C}^{m \times P}$ represents the partially sampled Fourier encoding matrix.

2.1. CSMRI

CSMRI reconstructs the unknown x from the k-space measurements f. In a mathematical model, CSMRI solves the linear equations $F_u x = f$ by minimizing the l_0 quasi norm of the sparsified image Ψx, where Ψ represents a global and typically orthonormal sparsifying transform. The corresponding optimization problem is

$$\min_x \|\Psi x\|_0 \quad s.\ t. \quad F_u x = f \tag{1}$$

Eq. (1) means that we should find a sparse code x for the given f using the matrix F_u, and this problem is NP-hard. It can be solved by greedy algorithms, such as orthogonal matching pursuit (OMP) [32,33]. The CSMRI reconstruction problem always uses l_1 relaxation of the l_0 quasi norm, and accounts for the noise in the k-space measurements in the following objective function [3].

$$\min_x \|F_u x - f\|_2^2 + \lambda \|\Psi x\|_1 \tag{2}$$

Because MR images are typically non-stationary, there are not universal domains in which all parts of the images are sparse. The predefined sparsifying transforms are not able to efficiently represent a specific signal and they are lack of the adaptivity to the image local structures. Recently, in medical image reconstruction field, Ravishankar et al. [10] present a method named DLMRI, which uses adaptive dictionary to reconstruct MR image by solving the following minimization problem

$$\min_{x,D,\Gamma} \sum_l \|R_l x - D\alpha_l\|_2^2 + \nu \|F_u x - f\|_2^2 \quad s.\ \mathrm{t.} \quad \|\alpha_{ij}\|_0 \le T_0 \ \ \forall\ l \tag{3}$$

where x denotes the reconstructed image, $\Gamma = [\alpha_1, \alpha_2, \cdots \alpha_L]$ denotes the sparse coefficient matrix of image patches, R_l represents the operator that extracts the patch from x, D stands for the adaptive dictionary, α_l denotes the redundant sparse representation of $R_l x$ over D, T_0 is a threshold of the sparsity. The regularization parameter ν balances the tradeoff between the two terms, and it is empirically chosen as $\nu = \theta/\sigma$, where σ is the standard derivation of the measurement noise and θ is a positive constant. By learning adaptive transforms from image, DLMRI has exhibited superior performance compared to those methods using fixed basis. The popular K-SVD method [18] can solve Eq. (3) effectively. However, DLMRI is a patch-based redundant sparse recovery method, which indicates that the overall image is reconstructed by averaging all the overlapped patches. Although it suppresses the artificial noise, the structure information of the image is also easy to be blurred with the square patches because of the lack of sparsity in isolated points or edges. To enhance the sparsity in complex regions and restrain the flaw in dictionary learning, we use the SADCT as a new regularization term to exploit image sparsity and preserve edges, which will be presented in Section 3.

2.2. Shape-adaptive DCT (SADCT)

In the last decade, many significant research works have been made to promote the development of shape-adaptive transforms. In this field, the main focus point is to construct a model that can be used efficiently for the analysis of arbitrarily shaped image segments, where the data always exhibit some uniform behavior. SADCT transform adaptively finds the unknown smooth and anisotropic region in an image patch, and then applies DCT transform to the found region. SADCT actually involves two independent problems: (1) the transform should adapt to the shape (i.e., a shape-adaptive transform) and (2) the shape itself must adapt to the image features (i.e., an image-adaptive shape). The first problem has found a very satisfactory solution in the SADCT transform. The SADCT transform is a cascaded application of DCT transform, and it is computed by one-dimensional varying-length DCT transform first on the columns and then on the rows of the anisotropic region. The SADCT does not require computation-expensive matrix inversions or iterative orthogonalizations. It can be interpreted as a direct generalization of the classical DCT transform. So, SADCT has a computational complexity comparable to that of the conventional separable block-DCT. Fig. 1 shows the process for realizing SADCT.

In this paper, the anisotropic Local Polynomial Approximation (LPA) -Intersection of Confidence Intervals (ICI) [26–28] is used to solve the second problem. LPA is a technique which is applied for