Physics Letters B 786 (2018) 11–15

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Quartet structure of $N = Z$ nuclei in a boson formalism: The case of ^{28}Si

M. Sambataro [a,*], N. Sandulescu [b]

[a] Istituto Nazionale di Fisica Nucleare – Sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy
[b] National Institute of Physics and Nuclear Engineering, P.O. Box MG-6, Magurele, Bucharest, Romania

ARTICLE INFO

Article history:
Received 26 July 2018
Received in revised form 29 August 2018
Accepted 4 September 2018
Available online 18 September 2018
Editor: J.-P. Blaizot

Keywords:
$N = Z$ nucleus
Quartets
IBM
U(5)–$\overline{\text{SU}(3)}$ transition

ABSTRACT

The structure of the $N = Z$ nucleus ^{28}Si is studied by resorting to an IBM-type formalism with s and d bosons representing isospin $T = 0$ and angular momentum $J = 0$ and $J = 2$ quartets, respectively. $T = 0$ quartets are four-body correlated structures formed by two protons and two neutrons. The microscopic nature of the quartet bosons, meant as images of the fermionic quartets, is investigated by making use of a mapping procedure and is supported by the close resemblance between the phenomenological and microscopically derived Hamiltonians. The ground state band and two low-lying side bands, a β and a γ band, together with all known $E2$ transitions and quadrupole moments associated with these states are well reproduced by the model. An analysis of the potential energy surface places ^{28}Si, only known case so far, at the critical point of the U(5)–$\overline{\text{SU}(3)}$ transition of the IBM structural diagram.

1. Introduction

The important role played by quartets in $N = Z$ nuclei has been known for a long time [1–6]. By quartets we denote here alpha-like four-body correlated structures formed by two protons and two neutrons coupled to total isospin $T = 0$. Recently, microscopic quartet models have been successfully employed to describe the proton–neutron pairing [7–12] as well as general two-body interactions [13–16] in $N = Z$ nuclei. As a basic outcome, the $J = 0$ quartet has been found to play a leading role but other low-J quartets have also been found essential to describe the spectra of $N = Z$ nuclei.

The difficulties associated with a microscopic treatment of $N = Z$ nuclei in a formalism of quartets rapidly grow with increasing the number of active nucleons. To make the application of this formalism possible also for large systems, in the present work we propose an approach where elementary bosons replace quartets. Based upon the above fermionic studies, we search for a description of $N = Z$ nuclei in terms of only two building blocks, the $T = 0$, $J = 0$ and $T = 0$, $J = 2$ quartets. These quartets are therefore represented as elementary s and d bosons, respectively. This bosonic architecture clearly coincides with that of the Interacting Boson Model (IBM) in its simplest version [17]. The application of this model, in terms of quartets, to $N = Z$ nuclei is, however, without precedent. We remark that in the standard IBM framework a proper treatment of even–even $N = Z$ nuclei would imply the use of the much more elaborate IBM-4 version of the model [18] which carries 10 different types of pair bosons. We also notice that an IBM-type approach based on quartet bosons was applied long ago [19], on a purely phenomenological basis, to nuclei with protons and neutrons occupying different major shells, i.e. nuclei which are commonly described by IBM-2 [17].

The manuscript is structured as follows. In Section 2, we illustrate our formalism. In Section 3, this formalism is applied to a description of ^{28}Si. In Section 4, we discuss the geometric structure of this nucleus. Finally, in Section 5, we give the conclusions.

2. The formalism

We start by setting the general quartet boson formalism for the treatment of $N = Z$ nuclei. We describe these nuclei in terms of collective $T = 0$ ($J = 0$ and 2) quartets that we represent as elementary sd bosons. By denoting the corresponding boson creation operators as $b_0^\dagger = s^\dagger$ and $b_{2\mu}^\dagger = d_\mu^\dagger$ (μ being the angular momentum projection), the most general one-body plus two-body Hamiltonian takes the standard IBM form

* Corresponding author.
E-mail addresses: michelangelo.sambataro@ct.infn.it (M. Sambataro), sandulescu@theory.nipne.ro (N. Sandulescu).

https://doi.org/10.1016/j.physletb.2018.09.011

$$H_B = \sum_{\lambda} \epsilon(\lambda)\hat{n}_{\lambda} + \sum_{\lambda_1\lambda_2,\lambda_1'\lambda_2',\Lambda} V(\lambda_1\lambda_2,\lambda_1'\lambda_2';\Lambda)[[b^{\dagger}_{\lambda_1} b^{\dagger}_{\lambda_2}]^{\Lambda}[\tilde{b}_{\lambda_1'}\tilde{b}_{\lambda_2'}]^{\Lambda}]^0 \quad (1)$$

where $\hat{n}_{\lambda} = \sum_{\mu} b^{\dagger}_{\lambda\mu} b_{\lambda\mu}$ and $\tilde{b}_{\lambda\mu} = (-1)^{\lambda+\mu} b_{\lambda-\mu}$. To evaluate to what extent the quartet bosons can be associated to microscopic quartets as well as to have an initial guess for the parameters of this Hamiltonian we shall resort to a mapping procedure. Mapping procedures allow to establish a link between spaces of composite and elementary objects and have been largely employed in a microscopic analysis of the IBM [20]. In this work we will follow the general lines of the procedure of Ref. [21] adapted for the quartet case.

We begin by introducing the most general quartet with isospin $T = 0$ and angular momentum (projection) $J(M)$

$$Q^{\dagger}_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} C^{(J)}_{i_1 j_1 J_1, i_2 j_2 J_2, T'} \times \left[[a^{\dagger}_{i_1} a^{\dagger}_{j_1}]^{J_1 T'} [a^{\dagger}_{i_2} a^{\dagger}_{j_2}]^{J_2 T'} \right]^{J,T=0}_{M}. \quad (2)$$

With N such quartets we construct the fermionic quartet space

$$F^{(N)} = \{Q^{\dagger}_{i_1} Q^{\dagger}_{i_2} \cdots Q^{\dagger}_{i_N}|0\rangle\}_{i_1 \le i_2 \cdots \le i_N}, \quad (3)$$

where $Q^{\dagger}_i \equiv Q^{\dagger}_{J_i M_i}$. To the quartet operator $Q^{\dagger}_i$ we associate the boson $b^{\dagger}_i$ and, in correspondence with the fermion space $F^{(N)}$, we define the boson space

$$B^{(N)} = (\mathcal{N}_{i_1 i_2 \ldots i_N})^{-1/2} b^{\dagger}_{i_1} b^{\dagger}_{i_2} \cdots b^{\dagger}_{i_N}|0\rangle\}_{i_1 \le i_2 \cdots \le i_N}, \quad (4)$$

where $\mathcal{N}_{i_1 i_2 \ldots i_N}$ is a normalization factor. There is a one-to-one correspondence between the states of $F^{(N)}$ and $B^{(N)}$, the basic difference being that the boson states are orthonormal while the fermion ones are not. In correspondence with a fermion Hamiltonian H_F, we define a boson hamiltonian H_B such that

$$(N,l|H_B|N,m) = \sum_{ij} R^{(N)}_{li} \langle N,i|H_F|N,j\rangle R^{(N)}_{jm} \quad (5)$$

where $|N,i\rangle$ and $|N,i)$ are generic states of $F^{(N)}$ and $B^{(N)}$, respectively, and $R^{(N)}_{li} = \sum_k {}^* f^{(N)}_{lk} \mathcal{N}^{(N)-1/2}_k f^{(N)}_{ik}$ with $f^{(N)}_{lk}$ and $\mathcal{N}^{(N)}_k$ being the eigenfunctions and eigenvalues of the overlap matrix of the fermion states $|N,i\rangle$, respectively. The asterisk in the expression for $R^{(N)}_{li}$ means that the sum is extended only over the non-zero eigenvalues $\mathcal{N}^{(N)}_k$. It can be proved that the eigenspectrum of H_B contains all the eigenvalues of H_F in $F^{(N)}$ plus a number of zero's corresponding to the states with $\mathcal{N}^{(N)}_k = 0$. The boson Hamiltonian H_B so constructed is Hermitian and, in general, N-body. Analogous expressions for H_B, but for pair bosons, can be found in Refs. [22,23].

3. The spectrum of ^{28}Si

We apply the formalism just described to the nucleus ^{28}Si. ^{28}Si has 6 protons and 6 neutrons outside the ^{16}O core. Thus we describe this nucleus in terms of three collective quartets that we represent as elementary *sd* bosons. The corresponding theoretical spectrum has only 10 states. The angular momenta of the states are such that these can be arranged into a ground state band and two side bands, a β and a γ band. Correspondingly, as experimental spectrum of ^{28}Si we consider only the ground state band and two low-lying β and γ bands. These β and γ bands have their

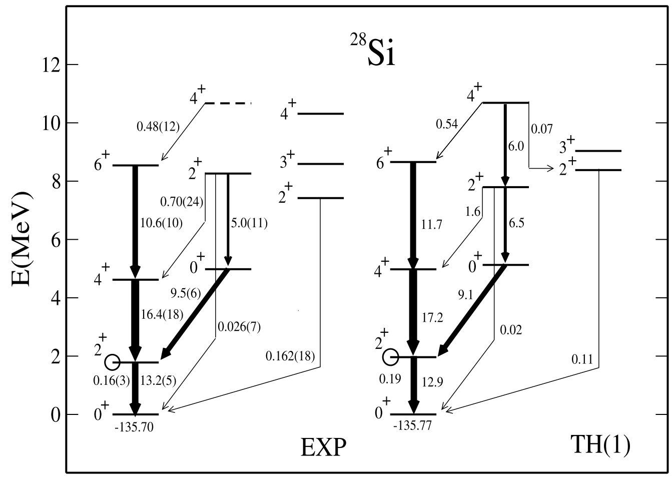

Fig. 1. Experimental [26] and theoretical low-energy spectra of ^{28}Si. Arrows represent B(E2) transitions and the corresponding values (in W.u.) are given by the numbers next to them. The circle on the 2^+ level stands for the quadrupole moment of this state (in eb). The number below the ground state gives the binding energy (experimental value from Ref. [37]).

band heads at 4.98 MeV and 7.42 MeV, respectively. According to Ref. [24], these ground, β and γ bands share a common intrinsic structure, all being classified as "oblate". These experimental bands are shown on the left side of Fig. 1. Some uncertainties are present for the $J = 4$ state of the β band due to the lack of experimental information. The state which has been tentatively inserted in Fig. 1 is the $J = 4$ state at $E = 10.67$ MeV. It is worth mentioning that the experimental spectrum shown in Fig. 1 is only a part of the complex spectrum of ^{28}Si, which contains many other bands [24].

To describe the spectrum of Fig. 1 with the Hamiltonian (1) we proceed as follows. As quartets $Q^{\dagger}_{JM}$ (2) we define those which result from a diagonalization of the USDB Hamiltonian [25] in a space of two protons and two neutrons coupled to $T = 0$ and $J = 0, 2$. According to the mapping procedure outlined in Section 2, the values of the single boson energies in the Hamiltonian (1) are therefore the energies of these quartets. These are precisely $\epsilon(0) = -37.713$ MeV for $J = 0$ and $\epsilon(2) = -36.158$ MeV for $J = 2$. The remaining parameters, i.e. the two-body matrix elements of the Hamiltonian, are fitted to the experimental data. As a starting point for this fit we have used the two-body matrix elements derived from the USDB interaction according to the boson mapping presented above. These matrix elements are shown in Fig. 2 (dashed line). In the same figure we show (solid line) the matrix elements which provide the best fit of the experimental spectrum. With the notation of Fig. 2, the adopted values are (in MeV): (1) = -3.374, (2) = -0.859, (3) = -3.875, (4) = -14.298, (5) = -2.348, (6) = -6.746, (7) = -9.316. Some differences can be seen between microscopically derived and phenomenologically fitted parameters (particularly at point (3)). These differences, which have significant effects on the final spectrum, are expected to reflect a renormalization of the boson parameters which takes into account the lack of $J > 2$ quartets (whose role has been previously pointed out [13]) as well as the lack of three-body terms in H_B. The overall agreement between the two set of parameters of Fig. 2 is, however, such to support the microscopic interpretation of the bosons as images of $T = 0$ quartets.

The theoretical spectrum of the Hamiltonian (1) with the parameters fitted as discussed above is shown Fig. 1. A good agreement is seen between theory and experiment. The calculations generate also a 0^+ state at 11.61 MeV, not shown in the figure. The only certain experimental 0^+ state present in the ENSDF database [26] in this region is located at 10.27 MeV but it is a $T = 1$ state.