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The fate of black hole horizons in semiclassical gravity

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ABSTRACT

The presence of a horizon is the principal marker for black holes as they appear in the classical theory of gravity. In General Relativity (GR), horizons have several defining properties. First, there exists a static spherically symmetric solution to vacuum Einstein equations which possesses a horizon defined as a null-surface on which the time-like Killing vector becomes null. Second, in GR, a co-dimension two sphere of minimal area is necessarily a horizon. On a quantum level, the classical gravitational action is supplemented by the quantum effective action obtained by integrating out the quantum fields propagating on a classical background. In this note we consider the case when the quantum fields are conformal and perform a certain non-perturbative analysis of the semiclassical equations obtained by varying the complete gravitational action. We show that, for these equations, both of the above aspects do not hold. More precisely, we prove that i) a static spherically symmetric metric that would describe a horizon with a finite Hawking temperature is, generically, *not* a solution; ii) a minimal 2-sphere is *not* a horizon but a tiny throat of a wormhole. We find certain bounds on the norm of the Killing vector at the throat and show that it is, while non-zero, an exponentially small function of the Bekenstein–Hawking (BH) entropy of the classical black hole. We also find that the possible temperature of the semiclassical geometry is exponentially small for large black holes. These findings suggest that a black hole in the classical theory can be viewed as a certain (singular) limit of the semiclassical wormhole geometry. We discuss the possible implications of our results.

1. Introduction

Existence of black holes is one of the most fascinating predictions of Einstein's theory of gravity. There is accumulating astrophysical evidence that black holes, or compact objects that look pretty much like black holes, are not rare in the Universe. The catalog [1] of stellar-mass black holes contains hundreds of candidates. Supermassive black holes are believed to be in the center of any galaxy including the Milky Way. Yet another evidence comes from the recent detection of a gravitational wave signal which originates from a coalescence of two massive black holes [2]. However, the direct detection of a black hole event horizon remains the principal experimental challenge.

On the other hand, there have been suggestions [3], [4], [5], [6], [7] that wormholes may mimic very closely the behavior of black holes, including the geodesics and the characteristic quasi-normal modes, although not having the defining property of black holes – the existence of a horizon. The latter is replaced by a tiny throat, the longitudinal size of which is such that it may serve as a storage for the information ever fallen into the "black hole". For sufficiently small deviation parameter, the wormhole geometry is very difficult, if ever possible, to distinguish experimentally from the black hole geometry. In particular, as was discussed in [8], the gravitational ringdown encoded in the shape of the recently observed gravitational wave signal is not sufficient to actually probe the horizon and, thus, distinguish the two geometries.

The goal of this note is to provide more evidence for the wormhole picture and to demonstrate that, on the theoretical side, the existence of black holes, as we know them in General Relativity, is far from evident as soon as the quantum modifications of GR are taken into account. Indeed, the quantum fields generate a certain, generally non-local, modification (see for instance [9]) of the gravitational action and a respective modification of the gravitational equations. In a semiclassical description, in which the gravitational field (metric) is not quantized and all other matter fields are considered to be quantum, the modified Einstein equations de-

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fine the so-called semiclassical gravity. These modified equations, being fundamentally non-local, are extremely complicated so that the exact solutions can be found only in some very special, symmetric, cases [10], [11]. Previous works on semiclassical black holes include [12].

There are two aspects of classical horizons as they appear in GR. First of all, a horizon is simply a special surface in a static spherically symmetric metric on which the time-like Killing vector becomes null. The non-vanishing gradient of the norm of the Killing vector at the horizon defines the Hawking temperature. To leading order, near the horizon, the Einstein equations are satisfied for any temperature. The latter is fixed to be related to the mass by considering the solution globally, everywhere between the horizon and spatial infinity.

The second aspect relates horizons to surfaces of minimal area. Indeed, in GR if a co-dimension two sphere Σ is minimally embedded in a four-dimensional static spacetime then this sphere is a horizon (or, mathematically more rigorously, a bifurcation sphere of the event horizon). This second aspect is less known so that we will review it below.

Our main goal in this note is to analyze both these aspects in the framework of a semiclassical theory of gravity. To simplify the analysis we shall consider the quantum modification of the gravitational action produced by quantum conformal fields. In this case the scaling properties of the quantum action are uniquely fixed by the conformal charges of the CFT. This helps to make a rather general analysis for an arbitrary unitary CFT. Yet, our analysis is essentially local: we expand the metric in a small vicinity of the would be horizon and analyze the local solution to the modified gravitational equations. Thus, we do not have access to the global behavior of the solutions. However, this local analysis happens to be extremely informative as it allows us to rule out solutions with horizons, and, in fact, detect the drastic deviations from the classical behavior. More precisely, we have found that

i) a static spherically symmetric metric with a horizon characterized by a finite (non-vanishing) temperature is, generically, *not* a solution to the semiclassical gravitational equations[1];
ii) in semiclassical gravity, a 2-sphere of minimal area embedded in a static spacetime is *not* a horizon. Rather, it is a throat of a wormhole. We find a bound on the norm of the Killing vector at the throat and show that it is an exponentially small function of the Bekenstein–Hawking entropy of the classical black hole.

Thus, the static solutions to the semiclassical gravity are *horizonless* and the classical horizons are replaced by wormholes! This is as anticipated in [3]. Our result ii) shows that for the astrophysical black holes, the parameter (the smallest value of the norm of the Killing vector) that characterizes the deviation of the wormhole geometry from that of a black hole, although non-vanishing, is extremely small. That is why it might be extremely difficult to detect the deviation experimentally. Below we discuss this and other implications of our findings. We stress that our results concern only the static configurations. Although we anticipate that they can be extended to a stationary, rotating case, we can not exclude that there may exist some dynamical, time dependent solutions with an evolving horizon.

[1] This statement is not to be confused with the smooth horizon structure of the well-understood Hartle–Hawking state for Rindler spacetime or similar set-ups. As we comment in remarks (d) of section 7, we are exclusively looking at states (such as Boulware), where the stress-tensor does diverge near the horizon and as a result modifies the geometry there.

2. Two aspects of horizons in GR

We consider a static spherically symmetric metric of the general form (we prefer to work in the Euclidean signature)

$$\begin{aligned} ds^2 &= \Omega^2(z) g_{\mu\nu} dx^\mu dx^\nu \\ &= \Omega^2(z)\left(dt^2 + N^2(z)dz^2 + R^2(z)(d\theta^2 + \sin^2\theta\, d\phi^2)\right). \end{aligned} \tag{1}$$

The geometrical radius of a 2-sphere is $r(z) = R(z)\Omega(z)$. Upon varying the gravitational action with respect to $\Omega(z)$, $N(z)$ and $r(z)$ one gets three equations, one of which by Bianchi identities follows from the other two. The norm of the time-like Killing vector $\xi = \partial_t$ is $\xi^2 = \Omega^2(z)$. Vanishing of this norm signals the existence of a horizon. Clearly, this is a point (or in fact a 2-sphere) where the function $\Omega(z)$ vanishes. A particular choice for the function $N(z)$ is a matter of convenience and a choice of the coordinate system.

I. Universality near horizon. Consider the gauge $N(z) = 1$. Assuming that there exists a horizon at $r = r_h$ with a finite temperature $T = 1/\beta$, one finds the near horizon behavior

$$\Omega(z) = e^{-2\pi z/\beta} + \dots, \quad R(z) = r_h e^{2\pi z/\beta} + \dots, \tag{2}$$

where $\dots$ stand for subleading terms. The regularity of the metric requires the Euclidean time to be periodic with period β. Notice that in these coordinates the horizon is located at $z \to \infty$. In this regime, the optical metric $g_{\mu\nu}$ in (1) universally approaches [13] a product space of one-dimensional circle S_1^β with a 3-dimensional hyperbolic space H_3 of radius $\beta/(2\pi)$.

Horizon in classical theory (aspect A): *There exists an exact solution to the classical Einstein equations (with or without cosmological constant) such that the static spherically symmetric metric locally behaves as in (2).* To leading order, the equations are satisfied for any β. The latter is related to the mass by studying the solution globally.

II. Horizon as a minimal surface. Consider now the gauge $N(z) = 1/\Omega(z)$. In this case the radial coordinate $\rho = z$ measures the geodesic distance in the radial direction. The two independent Einstein equations then take the form (written in terms of geometrical radius $r(\rho)$)

$$\begin{aligned} &2rr'' + r'^2 - 1 = 0, \\ &\Omega(r'^2 - 1) + 2rr'\Omega' = 0. \end{aligned} \tag{3}$$

Horizon in classical theory (aspect B): *Suppose that the 2-sphere at $\rho = \rho_h$ is a minimal area surface, i.e. $r' = 0$ at $\rho = \rho_h$. Then it follows from the second equation in (3) that the function $\Omega(\rho_h) = 0$ and, hence, $\rho = \rho_h$ is a horizon.* Notice that $\Omega'(\rho_h)$ is not determined from (3), it is a constant of integration. Fixing the temperature as the periodicity in the Euclidean time t, we can determine $\Omega'(\rho_h)$ by the condition of absence of a conical singularity in the metric (1). Of course, globally, equations (3) describe nothing else but the Schwarzschild solution.

Below we shall examine the validity of the analogous aspects in the semiclassical gravity.

3. Semiclassical gravitational equations

The semiclassical gravitational action is composed by adding to the classical Einstein–Hilbert action $W_{EH}[G]$ a quantum effective action $\Gamma[G]$ obtained by integrating out the quantum matter fields. For simplicity we shall consider conformal fields. In