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Dynamic behavior of temperature field in a buoyancy-driven turbulent fire

Kazushi Takagi [a], Hiroshi Gotoda [a,*], Isao T. Tokuda [b], Takaya Miyano [b]

[a] *Department of Mechanical Engineering, Tokyo University of Science, 6-3-1 Niijuku, Katsushika-ku, Tokyo, 125-8585, Japan*
[b] *Department of Mechanical Engineering, Ritsumeikan University, 1-1-1 Nojihigashi, Kusatsu, Shiga 525-8577, Japan*

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ABSTRACT

We study the dynamic behavior of temperature field in a buoyancy-driven turbulent fire from the viewpoints of symbolic dynamics, complex networks, and statistical complexity. The permutation entropy and the horizontal visibility network entropy allow us to capture the subtle changes in temperature fluctuations. The possible existence of deterministic chaos in temperature fluctuations, as well as in streamwise flow velocity fluctuations [Takagi et al., Phys. Rev. E 96 (2017) 052223], is clearly verified using the multiscale complexity-entropy causality plane.

1. Introduction

Combustion is a well-known example of chemical systems with extremely high temperatures. A rich spectrum of flame front instabilities arises in an unsteady combusting flow mutually coupling with hydrodynamic, heat, and mass diffusion processes through a rapid chemical reaction, leading to the onset of chaotic dynamics during combustion. In open diffusion flames, the entire flow field mostly comprises of two gases: combustion products (low-density gas) behind the flame front and the surrounding ambient air (high-density gas). The interface between the two gases becomes unstable owing to the buoyancy-driven Kelvin–Helmholtz type instability mechanism, resulting in the formation of an upward toroidal vortex [1–3]. The toroidal vortex acts strongly on flame front so as to distort the flame shape, and gives rise to accompanying self-excited flame front oscillations with large-amplitude owing to the Rayleigh–Taylor instability mechanism [4]. These physical processes are strongly associated with the generation and growth of a buoyancy-driven turbulent fire [5]. We have recently conducted a numerical simulation of a buoyancy-driven turbulent fire and found the possible presence of two important dynamics in flow velocity field: low-dimensional deterministic chaos in the near field dominated by the motion of toroidal vortices, and high-dimensional chaos in the far field forming a well-developed turbulent plume [6]. The presence of these dynamics was identified from the viewpoints of symbolic dynamics, complex networks, and statistical complexity.

Nonlinear time series analysis has become increasingly prevalent in many fields of physics and chemistry, and has currently been used for understanding the complex dynamics appearing in various flame front and combustion instabilities [7–24]. Thus far, numerous experimental and numerical studies on a buoyant plume and pool fires have shown the mean/instantaneous flow velocity and temperature distributions [4,25–36], correlating the distinct oscillation frequency with the Froude number, Richardson number, and Strouhal number [4,25,27–30,32,37,38]. However, the characterization of the dynamic behavior of flow velocity and temperature fields using nonlinear time series analysis has not been conducted in the previous studies on buoyancy-driven turbulent fires. In particular, the nonlinear dynamics of temperature field remains to be delineated. A new approach using nonlinear time series analysis would provide a crucial step towards a better understanding and interpretation of complex fire dynamics.

The objective of this study is to reveal the spatiotemporal dynamics of temperature field in a buoyancy-driven turbulent fire using nonlinear time series analysis based on symbolic dynamics, complex networks, and statistical complexity. In this study, we estimate two classes of entropy, the permutation entropy [39]

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[*] Corresponding author.
E-mail address: gotoda@rs.tus.ac.jp (H. Gotoda).

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and the horizontal visibility network entropy [40], to quantify the randomness of dynamics. The former in terms of symbolic dynamics considers the probability distribution of possible existing rank order patterns in a time series, while the latter in terms of complex networks considers the probability distribution of possible degrees in the horizontal visibility graph consisting of vertices and edges. In relation to the permutation entropy, we estimate the number of forbidden patterns in the permutation spectrum [41] to test for the presence of nonlinear determinism during a buoyancy-driven turbulent fire. The utility of the permutation spectrum has been shown in recent combustion studies [19,23]. The multiscale complexity-entropy causality plane (CECP) [42] in terms of statistical complexity, which incorporates the variations in the delay time of the phase space, enables us to quantify the degree of dynamical complexity at different time scales and characterize dynamical states from deterministic chaotic processes to stochastic processes. In this study, we apply the multiscale CECP for temperature fluctuations from the near field to far field. This paper is organized in five parts. A brief description of the numerical simulation is given in Sec. 2. The methodological framework of nonlinear time series analysis is described in Sec. 3. We present the results and discussion in Sec. 4 and give a summary in Sec. 5.

2. Numerical simulations

Similarly to in our recent study [6], we numerically simulate the spatiotemporal dynamics of a buoyancy-driven turbulent fire employing a large-eddy simulation. In this study, we solve the following set of governing equations: the mass conservation equation, the momentum conservation equation, the energy conservation equation, and the chemical species equations (see ref. [6] for details). We also consider a global single-step irreversible chemical reaction, a mixture fraction combustion model, and a low-Mach-number flow assuming that the pressure field is decomposed into a background component, a hydrostatic component, and a flow-induced perturbation. The viscous stress in the momentum conservation equation for a large-eddy simulation is given by the Smagorinsky model. We solve the governing equations by adopting second-order finite differences for spatial derivatives and an explicit second-order predictor–corrector scheme for temporal derivatives. The finite volume method is adopted to solve the radiative term in the energy conservation equation. We set the computational domain in the x, y, and z directions to 2.0 m, 2.0 m, and 4.0 m, respectively. The total number of cells is 1024000 and a uniform grid is used. Methane gas is supplied from a square center area with dimensions of 1 m × 1 m. The temporal resolution in the numerical simulation is 1 ms, and nonlinear time series analysis is adopted for temperature fluctuations during 28 s after a sufficient amount of time has elapsed after the initial transient of the numerical simulation.

3. Methodological framework of nonlinear time series analysis

The Shannon entropy characterizes the dynamical randomness in a nonlinear system from the viewpoint of information theory, and is defined as the rate of production of information by Eq. (1).

$$s[\mathbf{P}] = -\sum_i p_i \log_2 p_i, \qquad \mathbf{P} = \{p_i; i = 1, 2, \ldots, M\}. \tag{1}$$

Here, s is the Shannon entropy, p_i is a discrete probability function, and M is the bin number. For a completely randomness process with the uniform probability distribution of $\mathbf{P} = (1/M, 1/M, \ldots, 1/M)$, $s[\mathbf{P}]$ takes the maximum value $s_{\max}$ $(= \log_2 M)$. The normalized Shannon entropy S $(= s/s_{\max})$ is from 0 to 1. In this study, we estimate the permutation entropy [39] on the basis of symbolic dynamic approach. The permutation entropy considers the probability distribution of rank order patterns in the components of the phase space vectors as p_i in Eq. (1). M corresponds to the number of π_i $(= D!)$. After counting the number of realizations of permutations $q(\pi_i)$ for all vectors in the D-dimensional phase space $\mathbf{T}(t) = (T'(t_i), T'(t_i + \tau), \ldots, T'(t_i + (D-1)\tau))$ with embedding delay time τ, we compute the permutation entropy $S_p[\mathbf{P}]$ normalized by the maximum permutation entropy $s_{p,\max}$ as

$$S_p[\mathbf{P}] = \frac{-\sum_{\pi_i} p(\pi_i) \log_2 p(\pi_i)}{s_{p,\max}}, \tag{2}$$

where $p(\pi_i) = q(\pi_i)/(N - (D-1)\tau)$, T' is temperature fluctuations, and N is the number of discrete data points of $T'(t)$. S_p increases with increasing randomness of the dynamics and takes a value of unity for a completely random process. On the basis of a recent study [41], D is set to 5 for the estimation of the permutation entropy.

The permutation spectrum test [41], which is a developed version of the BP methodology [39], can test for the presence of nonlinear determinism in complex dynamics. The central idea of the permutation spectrum test is to investigate whether or not forbidden patterns appear in ordinal sequences that are obtained by the symbolization of a time series. In this method, we first partition $T'(t)$ into subsets with length L. After they are symbolized into ordinal sequences on the basis of the BP methodology, we obtain the permutation spectrum consisting of the frequency distribution of ordinal patterns for each subset and their standard deviation between the subsets. If nonlinear determinism is present in temperature fluctuations, zero standard deviation with some forbidden patterns (original patterns that are absent in the frequency distribution) will appear in the permutation spectrum. In contrast, there will be nonzero standard deviation and no forbidden patterns if stochastic process strongly dominates temperature fluctuations. In this study, L is set to 1000 $(= 1$ s$)$ so that the data number of $T'(t)$ is much larger than $D!$ [39].

The concept of the horizontal visibility graph [40], which is proposed as a simplified version of the natural visibility graph [43], serves as a bridge between nonlinear dynamics, graph theory, and time series analysis by transforming a time series into a graph with vertices under a geometrical criterion. This transformation has recently been used for various fluid systems [18,44]. The horizontal visibility network entropy captures the nature of the high sensitivity to initial conditions and has been shown to have good performance for classifying dynamical states appearing in radiative-heat-loss-driven flame front instability [20], a falling thin-film flow [46], and double-diffusive instability [45]. Gotoda et al. [46] showed that the horizontal visibility network entropy is effective for capturing a significant transition from low-dimensional chaos to high-dimensional chaos and is linked to the permutation entropy. In a similar way to in previous studies [20,45,46], we estimate the randomness of network structures constructed from temperature fluctuations using the horizontal visibility network entropy S_n,

$$S_n = -\sum_k P(k) \ln P(k), \tag{3}$$

where $P(k)$ is the degree distribution of the horizontal visibility graph. Note that when mapping each datum of $T'(t)$ into a node in a horizontal visibility graph, two arbitrary nodes i and j in an associated graph are linked if the corresponding $T'(t_i)$ and $T'(t_j)$ are larger than $T'(t_n)$, $\forall n \in (t_i, t_j)$.

The disequilibrium-based statistical complexity allows us to quantify the dynamical complexity and was developed by Lamberti et al. [47] as the Jensen–Shannon statistical complexity C_{JS}