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Locality based wealth rule favors cooperation in costly public goods games

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ABSTRACT

Most previous investigations on spatial Public Goods Games (PGGs) assume that individuals are self-organized and all are engaged in every available groups, which is in sharp contrast with realistic situations, where players may follow some local rules of whether they can participant or not. In this paper we embed the locality based wealth rule into self-adaptive region based mechanism to study the evolution of cooperation. When deciding about participation, both individual wealth of a player and qualification threshold of its local region are considered. Player whose wealth exceeds its local limit is allowed to participant in the joint venture by paying some entrance fee, which is independent of its strategy of whether cooperate or not. Otherwise, they are forbidden in the current round. Our result shows that the proposed mechanism can boost the emergence and maintenance of cooperation, with moderate values of controlling strength, and when the entrance fee is not harsh. Furthermore, small region segmentation scheme favors cooperation better with small controlling factor, whereas larger regional size can slow down the extinction process of cooperators when controlling factor is relatively high. We hope that our findings could shed light on better understanding of the emergence of cooperation among structured populations.

1. Introduction

Cooperation in real world situation can solve problems like global warming, overfishing and natural resources preservation [1–5]. However, how cooperation emerges and evolves among rational individuals remains unclear. Evolutionary game theory provides a powerful framework to investigate the problem of cooperation, and many efforts have been devoted to clarify why cooperators should pay some cost to benefit others [6–13].

To solve the rising dilemma between involving individual and collective benefit, one promising avenue is to enforce economic policy over the population. Studies on wealth heterogeneity reveals that diversity of economic means favors cooperation in structured populations. By varying the amount of investment across different cooperators [14,15] on the investment graph, or enforcing diverse payoff [16–18] over the distribution network, less resource is facing the danger of being exploited, because of network reciprocity that encourages poor individuals to learn from their rich counterparts. Cooperation also emerges when the investment or distribution diversity degenerates to a choice question between 1 and 0 [19–21]. When some of the individuals are not engaged in the joint venture, cooperators may have the chance to escape from being exploited due to the reduction of public resource, and defection may become less intensive as some defectors are forced to withdraw from the game.

Escaping from the game is possible for cooperators using conditional participation scheme. For example, voluntary participation [22–24] can offer an escape hatch out for some social traps, by introducing a third role, the loners. However, the additional loners cannot stabilize the dynamics and payoff of the loners remains equally the same. To address this problem, conditional cooperation [25–27] where cooperators can decide whether to denote or not is proposed. Furthermore, cooperation can also emerge using mixed strategies, by combining different forms of voluntary participation and conditional cooperation [28–30].

Social exclusion provide another avenue to reduce exploitation from defectors, by excluding low payoff groups [31], or adding new roles of excluders [32]. Social exclusion can also be considered an extreme case upon punishment [33]. On deciding which is more effective, imposing a fine on the defector's payoff, or preventing them to join the game, Liu et al. [34] found that social exclusion is better than punishment.

The work that is most related to ours [35] simulated the idea of probabilistic participation and conditional investment on spatial public goods games. A player is offered a chance to participant the game if his wealth exceeds a predefined threshold value, with a certain probability. The combination of wealth based rule and con-

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ditional participation mode is reported to favor cooperation better than pure probabilistic participation, where a predefined global value is utilized when deciding about participation. We argue that the single threshold selection scheme is rude and inadequate to describe choices of the entire population. It is still unclear who is supposed to be selected or eliminated in a more realistic scene. Existing work treats all individuals equally and ignored their spatial diversity. In a realistic situation, people from different regions may follow different criterion of the game. We quote house purchasing in real world as an example. As house prices vary across regions, whether one can afford a house depends mainly on individual wealth he possess and the local house price. Based on this observation, we propose a novel thresholding mechanism where different regions enforce different criterions, according to the local "house price" level. If an individual's wealth exceeds the corresponding regional threshold, he is allowed to participate in the group effort. Otherwise, the player is disqualified and is not supposed to play in the joint venture.

In a typical public goods game, up to N players can choose whether to cooperate or defect. Cooperators contribute a fixed amount c, while defectors invest none. The total contribution in the public pool is multiplied by an enhancement factor r and then equally distributed to all of the N participants. Hence, each defector would get an amount of rtc/N providing t out of N players choose to cooperate, while that for a cooperator should be $rtc/N - c$. We modify the classical model to a costly public goods game, where all participants should pay an additional participation fee independent of their strategies [36,37]. This is inspired by real life experiences when we loan some money from the bank, and we are required to mortgage some of our properties. As we will show, even when enforcing some extra fee, our locality based wealth rule can promote the emergence of cooperation better than the single threshold rule.

The remaining paper is arranged as follows. The proposed locality based model is shown in Section 2. Section 3 presents our simulation results and relevant discussions. And Section 4 shows theoretical analysis in well-mixed populations, which illustrates how the role of non-participants play in promoting cooperation. Concluding remarks are drawn in the end.

2. Model

Consider a population of size $M \times M$ located on a squared lattice, where each individual chooses a strategy of either cooperate or defect. According to the underlying topology, each individual in the population forms a group of size $k = 5$ centered at the focal individual of a von Neuman neighborhood and four of its nearest neighbors. In this model, not all individuals in the group are allowed to participant in the joint venture, and the locality based wealth rule is used to decide qualification of participation for each individual.

According to our region segmentation scheme, we first uniformly segment the population into non-overlapping patches of size $b \times b$. Suppose μ_i is the average payoff collected from region i, and σ_i standard deviation of that region, both are obtained from previous round of the game. We define $th_i = \mu_i + p\sigma_i$ the threshold whether player x within region i is allowed to participate the game. Player x is qualified if its payoff from the previous round exceeds th_i, which is independent of its strategy of whether cooperate or not. Otherwise, x is not allowed to play the game. Note that p is the controlling factor indicating qualification level of entering the game. Low negative values of p indicate low qualification level that the majority of the population is qualified, and high positive values verse vice. Besides, it is worth noticing that $p = 0$ indicates the case where regional threshold is equivalent to average wealth level of that region.

At each time step, all qualified players have to pay some participation fee g to each of the k groups they participate in before the game. Whenever playing the game, each participating cooperator should invest a fixed amount of c into the public goods, while the participating defectors don't. The total contribution is multiplied by an enhancement factor r and equally distributed to all the participants. Thus, the payoff for a qualified participant x in the group centered at player j is

$$\Pi_x^j = \frac{rN_{qc}^j c}{N_{qp}^j} - g - s_x c \tag{1}$$

where N_{qp}^j and N_{qc}^j are the number of qualified players and qualified cooperators out of j's k group members respectively. The symbol s_x denotes strategy of player x, $s_x = 1$ for cooperating and $s_x = 0$ for defecting. Note that if x is excluded from the game, its payoff should always be 0. Besides, the number of qualified cooperators should always be equal or less than the number of qualified players such that $N_{qc}^j \leq N_{qp}^j$. One special case when there is no participant in the group that $N_{qc}^j = N_{qp}^j = 0$, no interaction occurs and nothing obtains. Otherwise, when $N_{qp}^j \neq 0$, two extreme cases show up. When all participants are defectors that $N_{qc}^j = 0$, each participant in the group would end up getting a negative amount $-g$. On the other extreme when all participants invest into the pool that $N_{qp}^j = N_{qc}^j$, each participant obtains $(r-1)c - g$, which is expected to be a large positive value. Thus, dilemma exists between participating cooperators and defectors, denoted by the ratio of cooperation among all qualified players in the group. As participant x plays in k of its neighboring groups, its total payoff within one round of the game Π_x is accumulated from k related payoffs.

After accumulating payoff, all individuals would experience strategy updating. Individual x would adopt the strategy of individual y, who is chosen at random among all of x's neighbors, with a probability q according to Fermi rule that

$$q = \frac{1}{1 + exp[\beta(\Pi_x - \Pi_y)]} \tag{2}$$

where parameter β defines selection strength, measuring how important payoff is in deciding strategy updating [38].

We measure fraction of cooperators as well as fraction of qualified cooperators when the evolutionary stable state is reached. Equilibrium is detected when cooperation level for the last 200 iterations stay unchanged or the total number of iteration reaches 100,000. Each evolution is repeated 30 times for better statistics of the final result.

3. Results

We first present how controlling factor p influences cooperation level. Fig. 1(a) shows the fraction of cooperation who actually engaged in the game as a function of p at equilibrium state. Obviously, cooperation emerges and dominates with moderate values of p. The principle for this phenomenon is analogy to probabilistic participation [40] which was reported to favor cooperation [20,41]. For small values of p, majority of the population are qualified as the qualification threshold is far below the average level. Defectors can quickly invade into the population as performing deceptively provides them a route to plunder the public resources, hence more profitable than their cooperating counterparts. They can easily break cooperating clusters, sustaining and spreading their defective strategies to the rest of the population. However, as p gradually increases, it is more difficult for defectors to wipe out cooperators, as there remains fewer contributors as well as less public resource for them to exploit. Although intuitively, cooperators may earn even less than defectors for devoting contribution, their payoff would surpass their defecting counterparts when they form