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Conservation of a predator species in SIS prey-predator system using optimal taxation policy

Nishant Juneja[a,*], Kulbhushan Agnihotri[b]

[a] Research Scholar, I.K. Gujral Punjab Technical University, Kapurthala, Punjab, India

[b] Department of Applied Science and Humanities, Shaheed Bhagat Singh State Technical Campus, Ferozepur, Punjab, India

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ABSTRACT

In this paper, we present and analyze a prey-predator system, in which prey species can be infected with some disease. The model presented in this paper is motivated from D. Mukherjee's model in which he has considered an SI model for the prey species. There are substantial evidences that infected individuals have the ability to recover from the disease if vaccinated/ treated properly. In this regard, Mukherjee's model is modified by considering SIS model for prey species. Theoretical and numerical simulations show that the recovery of infected prey species plays a crucial role in eliminating the limit cycle oscillations and thus making the interior equilibrium point stable. The possibility of Hopf bifurcation around non zero equilibrium point using the recovery rate as a bifurcation parameter, is discussed. Further, the model is extended by incorporating the harvesting of predator population. A monitory agency has been introduced which monitors the exploitation of resources by implementing certain taxes for each unit biomass of the predator population. The main purpose of the present research is to explore the effect of recovery rate of prey on the dynamics of the system and to optimize the total economical net profits from harvesting of predator species, taking taxation as control parameter.

1. Introduction

The study of prey-predator interactions is the most dominant field of study, both in Environmental Science and in Applied Mathematics. Recently, numerous researchers have concentrated on many valuable and commanding mathematical models of infectious diseases in prey-predator system [1–4]. They have observed that not only the disease in the system affects the dynamics of the prey- predator population, but the prey-predator interactions also affect the dynamics of disease [5–8]. Some studies deals with the situation where only prey population has been infected with a disease [9–11], whereas some considers the infection in predator species only [12–14]. Few researchers have modeled the situation where both prey and predator species are infected with some disease [15–17]. Juneja and Agnihotri [18] modeled a situation in which only predator is infected with some disease. They found that the reduced predation rate of infected predator is helpful in making the system infection free. Peterson and Page [19], have modeled the wolf (predator) and moose (prey) species assuming that predator species eat only the infected prey because the infected species are low in their mobility and are easily cacheable. They have shown that wolf often kills the moose during predation when the moose is badly suffered with an ailment named 'echinococcus granulosus'. This fact is also justified by Chattopadhyay et al. [20], where they have shown that the Tilapia fishes when get infected with vivrio class of bacteria tends to come closer to the surface of sea and become an easily available food for their predators, the Pelican birds. So, it has been justified in all the above cited works that disease in the system plays an important role in the dynamics of the system.

Keeping in view the above fact, Mukherjee [21] has considered a three species eco-epidemiological model with Holling type-II functional response of predator. He finds out the conditions for the Hopf bifurcation around the non zero equilibrium point, with conversion efficiency of the predator as the bifurcation parameter. Many authors proposed and analyzed different mathematical models for prey-predator system, where either prey or predator or both of them are suffering from some disease, yet they ignored the fact that recovery of infective individuals from the disease is also possible. In an SIS model, susceptible individuals become infected after coming into contact with infective individuals. Infected individuals go back to the susceptible class following an infective period. Further, it is observed that if the infected moose species are vaccinated properly then they can recover from the disease. It is also

* Corresponding author.
E-mail addresses: rtmct2016@gmail.com (N. Juneja), agnihotri69@gmail.com (K. Agnihotri).

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noticed that instead of high mortality rate, some animals like roe and antelope, can still recover from foot-and-mouth infection [recovery reference]. So keeping in view the above discussed facts, we modified Mukherjee's model by incorporating recovery of the infected prey from the disease. The role of recovery rate in the dynamics of prey predator system is extensively shown in the paper.

Now a days, the increased human requirements for excess food and energy are the major causes of exploitation of natural resources, whereas, ecologists are paying much attention on the protection of environment. Keeping in view, these two differing approaches, many researchers are trying to find a sustainable advance technique in approximately all the fields of human activity. Several major instruments are recommended for choosing the appropriate control variable. Few of them can be, imposing the taxes and license cost, seasonal harvesting, etc. Harvesting of species usually has a remarkable impact on the dynamics of biological resources. The imposing of taxation is considered to be better as compared with other control measures because of its flexible nature. Earlier, Clark [22], considered a fishery situation in which taxation is taken as a control parameter. Thereafter, many such models were discussed [23,24]. Similar methodology has been adopted for multi-species [25,26].

The extended model deals with harvesting of predator species with constant harvesting effort. The aim of the present research is to search an accurate taxation policy that provides maximum feasible benefit via harvesting for the humanity as well as to preserve the resources. The existence and local stability of steady state have been discussed. Pontryagin Maximum principle is used for optimal tax policy.

2. Mathematical model

A prey-predator system consisting of three species namely Susceptible Prey (S), Infected or Diseased Prey (I) and Predator Species (P) in which the recovery of prey species from the infection is considered. It is assumed that the disease has spread among the prey population only and it is not genetically inherited. The predator population predates only the diseased prey. Let 'r' be the constant recruitment rate in prey species, d_1, d_2 and d_3 are the natural mortality rates of the vulnerable prey, diseased prey and the predator species respectively, 'γ' is the recovery rate of infected prey, 'β' is the incidence rate of disease in prey and 'c' is the conversion efficiency of predator species.

We also consider that predator species is harvested with effort rate E and catchability coefficient q. Further, a regulatory group monitors the management of the species by implementing a tax 'τ' for one unit biomass of predator species. The negative value of 'τ' shows the subsidy offered to the fisherman. The net financial income to the harvester (apparent rent) is $m[q(p-\tau)P-C]$, here 'p' is average market price per unit biomass of the predator population and 'C' is corresponding cost for unit effort indulged during harvesting. We suppose that the net investment rate is directly related to the 'apparent rent'. So we have

$$I = m[q(p-\tau)P - C] \tag{2.1}$$

where $0 \le m \le 1$. Eq. (2.1) shows that the optimal investment rate at particular instant is as same as the apparent rent (for $m = 1$) at that particular instant of time. If the apparent rent is negative, we take $m = 0$ i.e. when sensible disinvestment of capital stocks is not feasible. Now, if the harvesting is suffering from loss and disinvestment of stock is allowed at that instant, then the harvester gets profit by permitting a regular disinvestment of the capital stocks. For this, we take $I < 0$ and $m > 0$ as the negative investment denotes disinvestment.

Based upon above assumptions, following differential equations for the system are obtained:

$$\begin{aligned}
\frac{dS}{dt} &= r - \beta SI - d_1 S + \gamma I\\
\frac{dI}{dt} &= \beta SI - \frac{PI}{1+I} - d_2 I - \gamma I\\
\frac{dP}{dt} &= p\left(-d_3 + \frac{cI}{1+I}\right) - qEP\\
\frac{dE}{dt} &= lm(q(p-\tau)P - C)E - nE
\end{aligned}$$

where initial conditions are $S(0) = S_0, I(0) = I_0, P(0) = P_0, E(0) = E_0$.

The parameters used in the above model are positive for physically realistic population model. Now, we will discuss the dynamical properties of the model in both the cases i.e. in the absence of harvesting and in the presence of harvesting.

3. In the absence of harvesting

Firstly, we will discuss the dynamical properties of the model in the absence of harvesting i.e. $E = 0$. So, the above model equations take the form:

$$\begin{aligned}
\frac{dS}{dt} &= r - \beta SI - d_1 S + \gamma I\\
\frac{dI}{dt} &= \beta SI - \frac{PI}{1+I} - d_2 I - \gamma I\\
\frac{dP}{dt} &= p\left(-d_3 + \frac{cI}{1+I}\right)
\end{aligned} \tag{3.1}$$

3.1. Boundedness of solutions

Theorem 3.1. *All the solutions of the system (3.1) will be in the region:* $R = \left\{(S, I, P)\epsilon R_+^3 : 0 \le S + I + P \le \frac{r}{\lambda}\right\}$ *as* $t \to \infty$ *for* $\left(S(0), I(0), P(0)\right) \in R_+^3$, *where* $\lambda = \min(d_1, d_2, d_3)$.

Proof. Consider the function

$$W(t) = S(t) + I(t) + P(t)$$

So

$$\frac{dW}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dP}{dt}$$

i.e.

$$\frac{dW}{dt} = r - d_1 S - d_2 I - d_3 P + (c-1)\frac{PI}{1+I}$$

Now $c \le 1$ and choosing $\lambda = \min(d_1, d_2, d_3)$. We get

$$\frac{dW}{dt} \le r - \lambda W$$

By comparison theorem, $W(t) \le \frac{r}{\lambda}$ as $t \to \infty$

Hence the proof. □

3.2. Equilibria and stability analysis

The system (3.1) has three possible steady states

1. $\hat{E}(\hat{S}, 0, 0)$, *where* $\hat{S} = \frac{r}{d_1}$, which always exist.
2. $E'(S', I', 0)$, where $S' = \frac{d_2+\gamma}{\beta}$, $I' = \frac{r\beta - d_1(d_2+\gamma)}{d_2\beta}$, provided

$$\frac{\beta r - d_1 d_2}{d_1} > \gamma \tag{3.2}$$