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Effect of measuring noise on scaling characteristics in the dynamics of coupled chaotic systems

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ABSTRACT

We study the scaling features in the evolutionary dynamics of two coupled chaotic systems based on the sequences of return times into a Poincaré section, contaminated with additive (measuring) noise. Using three models of chaotic systems: the Rössler oscillator, the Lorenz system, and the nephron model, and the detrended fluctuation analysis (DFA) as an approach for data processing, we demonstrate that the anti-correlated sequences of return times of synchronous motions show a higher sensitivity to measuring noise than the positively correlated series of return times of asynchronous oscillations. This conclusion is confirmed by the results for various oscillatory regimes in all models considered.

1. Introduction

A measured time series always contains noise of various origins [1]. Even if the dynamics of the system under study is deterministic, the conversion of the analog signal to digital format is accompanied by rounding errors, which can be treated as measuring noise. This type of noise does not influence the underlying dynamics of the system; however, it affects the accuracy of the evaluation of signal characteristics and the reliability of the diagnosis of the system's state from experimental data. A generally used approach to the processing of noisy data is its pre-filtering that can significantly reduce noise impact, especially when the frequency ranges of the signal reflecting the system's dynamics and additive fluctuations do not overlap. In practice, such approach is typically used to remove high-frequency variations of a signal providing its smoothing, or to eliminate slow changes in the mean value treated as a trend for nonstationary time series. Over the past decades, filtering capabilities have been improved by means of wavelet-based techniques being able to extract localized fluctuations that are not removed with Fourier-based approaches [2–5].

Filtering does not always improve the characterization of noisy data sets. This is the case, e.g., when considering point processes, where information about the system's dynamics is encoded by the times of stereotype events, and the data set represents a sequence of time intervals between successive events [6]. Such a sequence is often a noise-like process, where external fluctuations are difficult to detect and eliminate. In this regard, the effect of additive noise on the characterization of the system's dynamics from measured data should be known for various types of complex processes. From general assumptions, it could be expected that a small noise would provide insignificant changes in signal characteristics in comparison with fluctuations of higher intensity. Nevertheless, the effect of relatively weak fluctuations can differ between signals of distinct complexity and various dynamical regimes.

In this study, we consider how measuring noise affects the scaling features of complex processes in the dynamics of two coupled chaotic oscillators characterized by the sequences of return times into a Poincaré section. Such systems demonstrate various entrainment forms, which depend on the coupling strength and the individual dynamics of the oscillators. Unlike the frameworks of the classical theory of synchronization for periodic oscillations that is accompanied by a locking of their frequencies or phases, chaotic synchronization includes a wider range of entrainment phenomena represented by full synchronization [7,8], phase synchronization [9,10], lag synchronization [11], and generalized synchronization [12–14]. The related entrainment influences the dynamics of individual oscillators and changes the scaling features of the return time sequences. In particular, chaotic synchronization typically reduces the degree of multifractality in these sequences [15], which can be treated as a kind of ordering appeared due to the coupling between interacting units. Besides changes in the multifractality, synchronous and asynchronous oscillations are often quantified by

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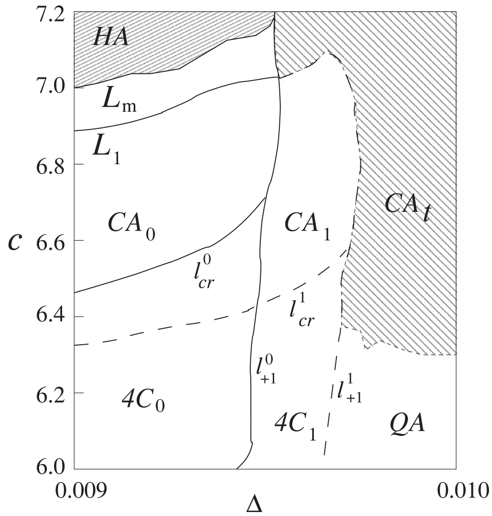

Fig. 1. A simplified bifurcation diagram of the model of two coupled Rössler oscillators.

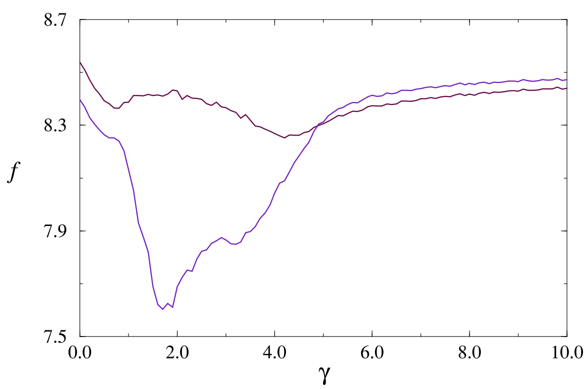

Fig. 2. Oscillation frequencies for two coupled Lorenz systems depending on the coupling strength.

different types of correlations, namely, by the anti-correlated sequences of return times related to synchronous dynamics and by positive correlations of return times for asynchronous regimes [15]. Distinctions in the structure of the return time sequences can exert different noise effect on the authentic characterization of the underlying dynamics from measured point processes.

Based on the detrended fluctuation analysis (DFA) [16–18], which is a widely used approach for studying the correlation properties of complex time series [19–24], we compare how measuring noise affects the error in characterizing the scaling features of various types of complex motions in the dynamics of interacting chaotic systems. We consider several models of chaotic oscillators including the Rössler system, the Lorenz oscillator and the nephron model, which exhibit oscillations with several different time scales, and show that the anti-correlated sequences of return times of synchronous motions demonstrate a higher sensitivity to measuring noise than the positively correlated series of return times of asynchronous oscillations. Such distinction in sensitivity to additive noise is confirmed for all models of coupled chaotic oscillators considered in this study.

2. Methods and models

2.1. Detrended fluctuation analysis

DFA is a variant of the correlation analysis of a data set, which is based on the transition to a random walk with its further root mean square analysis [16,17]. The algorithm includes the following four steps:

(1) The construction of a random walk $y(k)$ being a profile of the original data set $x(i)$, $i = 1, \dots, N$:

$$y(k) = \sum_{i=1}^{k} [x(i) - \langle x \rangle], \tag{1}$$

where $\langle x \rangle$ is the mean value.

(2) Segmentation of $y(k)$ into non-overlapping parts of fixed size n and linear fitting inside each part to obtain a piecewise linear function $y_n(k)$ that describes a local trend.

(3) Computing the root mean-square fluctuation $F(n)$

$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [y(k) - y_n(k)]^2}. \tag{2}$$

(4) Repeating the estimates for different n and computing the scaling exponent α, which describes the power-law behavior

$$F(n) \sim n^{\alpha}. \tag{3}$$

The value of α can be found as the slope of the dependence $F(n)$ in the log-log plot. This quantity relates to scaling exponents describing the behavior of the correlation function or the spectral power. For complex processes with a multiscale structure of data sets, $F(n)$ cannot be described by a simple power-law dependence with a single scaling exponent, and the local slopes of $\lg F$ vs. $\lg n$ vary depending on the size of the segment n. In this case, consideration of local scaling exponents seems preferable to a single quantity (global scaling exponent).

The values of $\alpha < 0.5$ quantify the anti-correlated statistics of the data samples $x(i)$, i.e. the alternation of large and small values of $x(i)$, when large values appear after small values and vice versa. Power-law correlations when large values mainly follow after large values and small values appear more often after small values are characterized by $\alpha > 0.5$. The uncorrelated dynamics of the data set is described by $\alpha = 0.5$.

2.2. Models of two coupled oscillators

2.2.1. Coupled Rössler systems

Two diffusively coupled Rössler oscillators represent a benchmark model of interacting nonlinear systems that produces a variety of complex dynamical regimes including regular, chaotic and hyperchaotic oscillations with different phase shifts between the signals of individual units. This model is described by six ordinary differential equations

$$\begin{aligned} \frac{dx_{1,2}}{dt} &= -\omega_{1,2} y_{1,2} - z_{1,2} + \gamma (x_{2,1} - x_{1,2}), \\ \frac{dy_{1,2}}{dt} &= \omega_{1,2} x_{1,2} + a y_{1,2}, \\ \frac{dz_{1,2}}{dt} &= b + z_{1,2}(x_{1,2} - c) \end{aligned} \tag{4}$$

The control parameters a, b and c define the dynamics of each system, and γ quantifies the coupling strength. The mismatch of the basic frequencies $\omega_1 = \omega_0 + \Delta$ and $\omega_2 = \omega_0 - \Delta$ provides nonidentical oscillations of the interacting units. Here, we used the following parameter set: a=0.15, b=0.2, γ=0.02, ω_0=1 and varied the parameters c and Δ to analyze the transitions to and between different types of chaotic attractors or to a hyperchaotic regime. The phenomenon of phase multistability in the model (4) was discussed, e.g., in [25], where the bifurcation mechanisms leading to the appearance of various attractors are described. A simplified bifurcation diagram showing the main dynamical regimes discussed