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An approach to improve the performance of fractional-order sinusoidal oscillators

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ABSTRACT

This paper presents a simple technique to approximate fractance devices (FDs) capable of improving the performance of any fractional-order oscillator. The proposed technique is based on an elementary mathematical tool of impedance equalization, and requires significantly lesser number of passive components than the existing FD approximation schemes. To compare the merit of approximated FDs with the existing *R-C* ladder based FDs, a well-known fractional-order Wien-bridge oscillator is realized using both FDs one by one; and the corresponding results are compared exhaustively. It is observed that the fractional-order oscillator realized using the proposed FDs gives better performance in terms of phase-noise, figure of merit (FoM), total harmonic distortion (THD), settling time, peak-to-peak voltage, power dissipation, and hardware compactness. Authenticity and accuracy of the proposed design has been verified using PSpice simulation and practical implementation.

1. Introduction

Fractional-order calculus is a branch of mathematics dealing with differentiation and integration of non-integer order, and often finds it applications in various areas of science and engineering [1–8]. Nowadays, concept of fractional calculus is being used in the designing of electronic oscillators [9]; such oscillators are known as fractional-order oscillators. These oscillators have some additional advantages over the conventional ones, such as précises phase and frequency control, wide range of oscillation frequency and more degree of freedom in system designing [10–18]. Ability to provide controlled phase shift, increases the popularity of fractional oscillators in several potential areas, for example communication system [19], medical science [20], musical instruments [21], etc. Fractional-order oscillators utilize fractance devices (FDs) in place of conventional capacitors and inductors. FDs are considered as fractional energy storing elements, and can be further categorized as fractional-order capacitor and fractional-order inductor. The current-voltage relations of fractional-order capacitor and inductor are defined as:

$$\text{Fractional} - \text{ordercapacitor}: \quad \frac{1}{C}i = \frac{d^\alpha v}{dt^\alpha} \tag{1}$$

$$\text{Fractional} - \text{orderinductor}: \quad \frac{1}{L}v = \frac{d^\alpha i}{dt^\alpha} \tag{2}$$

Unit of the fractional-order capacitor and inductor are $\mathrm{Fs}^{\alpha-1}$ and $\mathrm{Hs}^{\alpha-1}$ respectively, in place of F and H for the conventional elements. The dimensional formulae for fractional-order capacitor and fractional-order inductor are $[\mathrm{M}^{-1}\ \mathrm{L}^{-2}\mathrm{T}^{(3+\alpha)}\mathrm{I}^2]$ and $[\mathrm{M}\ \mathrm{L}^2\mathrm{T}^{(-3+\alpha)}\mathrm{I}^{-2}]$ respectively. Unfortunately, these fractional devices are not available commercially as lumped components; however several mathematical approximation schemes, such as continued fraction expansion, Regular newton process and Taylor series expansion, can be used to realize FDs using *R-C* tree or ladder circuits as shown in Fig. 1 [22–25].

Along with these passive realizations, some active realizations of the FDs are also available in literature [26]. One of the major drawbacks of these approximated FDs is the requirement of large number of passive components to realize a single FD. Some efforts have also been done to simulate single component FDs using various chemical processes [27,28], although they are not so popular in the area of electrical engineering. Consequently, the *R-C* tree or ladder network based FDs are still frequently used to design fractional-order oscillator.

Nowadays, power efficient and compact electronic devices are receiving great recognition. Unfortunately, existing fractional oscillators are neither power efficient nor compact, due to presence of tree or ladder network based FDs. Since the existing *R-C* network based FDs require large number of passive components; performance of fractional oscillators also degrade considerably in terms of noise, cost, and system reliability. Hence, there is a need to develop a technique to improve the energy efficiency, compactness and phase noise performance of any fractional-order oscillator.

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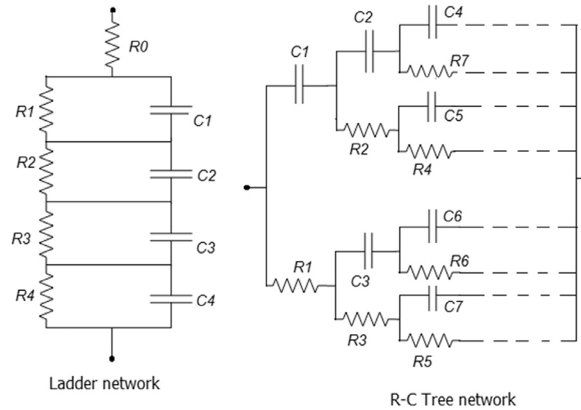

Fig. 1. Schematic of the tree and ladder network based FDs.

This paper is organized as follows: Section 2 presents general idea about the phase noise and figure of merit of an oscillator. New FD realization schemes suitable for fractional-order oscillators are proposed in Section 3. This section also describes the merits of the proposed FDs over the existing *R-C* network based FDs. Simulation results and implementations are discussed in Section 4. Finally, conclusions and future scopes are discussed in Section 5.

2. Noise in sinusoidal oscillators

For an electronic oscillator, noise can be categorised as amplitude noise and phase noise. A noisy signal $x(t)$ with amplitude A and angular frequency ω can be represented as:

$$x(t) = [A_o + A(t)]\sin\{\omega_o t + \varphi(t)\} \tag{3}$$

where $A(t)$ and $\varphi(t)$ are instantaneous amplitude and phase variation generated due to amplitude and phase noise respectively. In most of the cases the amplitude noise can be ignored, for example oscillators for providing a clock signal where the amplitude is clipped. On the other hand, the phase noise is closely associated with the jitter and will therefore have direct influence on the timing of the events, in clocked circuits [29]. Although in some applications, amplitude as well as phase noise are equally undesirable; phase noise creates greater mishap as it can be easily translated to amplitude noise. Unlike the excess phase $\varphi(t)$, the effect on the instantaneous amplitude $A(t)$ decays over time since oscillators have inherent amplitude restoring mechanisms in the oscillator circuit [30]. Hence phase noise is one of the major concerns in any oscillatory system as it changes the frequency spectrum as well as timing properties of oscillator. An ideal oscillator would have localized *Dirac impulse* tones at discrete frequencies whereas the spectrum of the perturbed oscillator is a *Lorentzian* at each harmonics [31, 32], and high power level at adjacent frequencies. Phase noise is measured in dBc/Hz, and defined in the form of single-sideband phase noise $L(f_m)$ as given below [33–34].

$$L(f) \triangleq 10\log_{10}\left(\frac{S_{ss}(f_o + f_m)}{P_{Total}}\right) \tag{4}$$

where $S_{ss}(f_o + f_m)$ is the oscillator power within 1 Hz bandwidth around offset frequency f_m from the central frequency f_o, and P_{Total} is the total power of the oscillator. It must be noted that the phase noise depends not only upon the oscillator design, but also upon the power dissipated and the offset frequency. Consequently, an additional parameter, figure of merit (FoM) is used to analyse the performance of an oscillator. The *FoM* normalizes the phase noise by power and oscillation frequency and hence improvement in *FoM* is truly a result of advancement in system designing, and not simply caused by feeding more power and frequency selection. The *FoM* is represented as [35]:

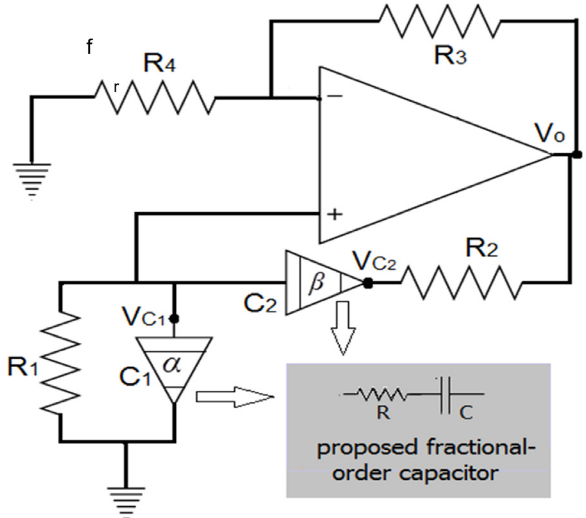

Fig. 2. Fractional-order Wien-bridge oscillator.

$$\mathrm{FoM} \triangleq |PN| + 20\log_{10}\left(\frac{f_o}{f_m}\right) - 10\log_{10}\left(\frac{P_{DC}}{1\ \mathrm{mW}}\right) \tag{5}$$

where f_o is oscillation frequency in *Hz, PN* is phase noise at f_m, and P_{DC} is the consumed power. To analyse the phase noise, an existing fractional-order Wein-bridge oscillator is taken into consideration [14]. This oscillator consists of four resistors R_1, R_2, R_3, R_4 and two fractional capacitors C_1, C_2 of order α and β respectively, as illustrated in Fig. 2.

Forward gain and the feed-back gain of the considered fractional-order Wien-bridge oscillator are given in (6) and (7) respectively.

$$A = 1 + \frac{R_3}{R_4} \tag{6}$$

$$B = \frac{R_1 s^{\beta} C_2}{R_1R_2C_1C_2 s^{(\alpha+\beta)} + R_1C_1s^{\alpha} + (R_1 + R_2)C_2s^{\beta} + 1} \tag{7}$$

Since the oscillatory circuits are arranged in positive feedback manner, the transfer function of the considered oscillatory system is expressed as:

$$\begin{aligned} H(s) &= \frac{A}{1 - AB} \\ &= A\left\{\frac{R_1R_2C_1C_2 s^{(\alpha+\beta)} + R_1C_1s^{\alpha} + (R_1 + R_2)C_2s^{\beta} + 1}{R_1R_2C_1C_2 s^{(\alpha+\beta)} + R_1C_1s^{\alpha} + (R_1 + R_2 - AR_1)C_2s^{\beta} + 1}\right\} \end{aligned} \tag{8}$$

The oscillators do not have any input terminal and they draw needful excitation from noise. In any electrical circuit *thermal noise* is everlasting since it is generated by the random motion of charge carriers. Behaviour of the *thermal* noise is *white Gaussian*, i.e. its spectrum is equally distributed with constant magnitude within the entire frequency band. This *white Gaussian* noise appears as input and provides needful excitation. Hence, the output of the system in *s*-domain will be identical to the system transfer function, i.e. $V_o(s) = H(s)$. For $R_1 = R_2 = R,\ C_1 = C_2 = C$ and $\alpha = \beta$, the frequency of oscillation (FO) and condition of oscillation (CO) are expressed as $\omega_o = (1/RC)^{1/\alpha}$ and $R_3/R_4 = 2\{1 + \cos(\alpha\pi/2)\}$ respectively [14]. The phase difference between the node voltages V_{C1} and V_{C2} is$\alpha\pi/4$. For $C_1 = C_2 = 1\ \mu\mathrm{F\ s}^{\alpha-1}$, $R_1 = R_2 = 10\ \mathrm{k}\Omega$ and $\alpha = \beta = 0.5$; the CO and FO are obtained as $R_3 = 3.14R_4$, and $\omega_o = 10,000$ rad/s i.e. $f_o = 1591.5$ Hz respectively. Now, the phase noise and *FoM* of the considered system is