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Analysis of dynamical behaviors of a 2-DOF vibro-impact system with dry friction☆

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ABSTRACT

In this paper, the discontinuous dynamics in a 2-DOF (two-degree-of-freedom) vibro-impact system with dry friction is investigated by using the flow switchability theory for discontinuous dynamical systems. Multiple domains and discontinuous boundaries are defined according to friction and impact discontinuity. Based on above domains and boundaries, the onset and disappearance conditions of sliding-stick motion are developed and the analytical conditions of side-stick motion and grazing motion are obtained mathematically. The switching sets and mapping structures are adopted to describe the complex motions in such discontinuous system. The numerical simulations are also given to illustrate the analytical results of motion switching on different boundaries. This investigation can help us understand the motion switching mechanism in non-stick, sliding-stick or side-stick motions of the oscillator with friction and impact.

1. Introduction

Impact and friction widely exist in mechanical engineering, and are the common and important contacts between two or more dynamical systems, and cause dynamical systems to be discontinuous. The researches on dynamical behaviors of the oscillator with impact and friction can help us better use or control them in practical problems. At present, scholars at home and abroad have done a lot of researches on dynamical behaviors of vibro-impact system. In 1982, Holmes [1] investigated the dynamics of repeated impacts with a sinusoidal vibrating table. In 1983, Shaw and Holmes [2] studied periodic motions, local bifurcations and chaotic motions of a SDOF (single degree of freedom) piecewise linear oscillator under a periodic force. Heiman et al. [3,4] discussed the periodic motions and stability of an inclined impact pair in 1987, and further investigated periodic motions and chaotic motions through bifurcation analysis and Poincare maps in 1988. In 1995, Han et al. [5] investigated periodic motions and chaotic motions of a horizontal impact pair and presented a parameter map of periodic motions. Through a combination of analytical, numerical, and experimental efforts, Long et al. [6] studied the non-smooth dynamics of an elastic structure excited by a harmonic impactor motion in 2008, and Chakraborty and Balachandran [7] investigated the non-smooth dynamics of impacting cantilevers at different scales in 2012. In these studies, dissipation is considered at the contact, and grazing impacts are considered, too. In 2015, Zhang and Fu [8,9] gave the analytical conditions to predict periodic motions in an inclined impact pair using discrete mapping theory of discontinuous dynamical systems, and further studied the dynamics of an inclined impact oscillator and obtained the analytical conditions of stick motion and grazing motion mathematically. In 2018, Xue and Fan [10] investigated discontinuous dynamical behaviors in a vibro-impact system with multiple constraints.

Friction contact between the surfaces of two objects is an important connection of transmission in mechanical engineering. In 1964, Filippov [11] investigated the discontinuous dynamical behaviors of a Coulomb friction oscillator, and presented a differential equation theory with discontinuous right-hand sides. The differential inclusion was introduced via the set-valued analysis for the sliding motion along the discontinuous boundary. In 1986, Shaw [12] discussed the stability for periodic motions of a friction-induced oscillator through Poincare mappings. In 1990, Awrejcewicz and Delfs [13] investigated the stability of equilibrium in a 2-DOF (two-degree-of-freedom) friction-induced oscillator. In 1994, chaotic dynamics of a dry-friction oscillator was investigated experimentally and numerically by Feeny and Moon [14]. In 1996, Oestreich et al. [15] presented the analysis of bifurcation and stability for a non-smooth friction oscillator. In 1998, using the variation of initial conditions, Natsiavas [16] gave the stability analysis for periodic motions of a class of harmonically excited SDOF oscillators with piecewise linear characteristics. An estimation was

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given for the steady-state responses in a Coulomb friction oscillator under a harmonic force by Hong and Liu [17] in 2000. In 2003, using computational method, Xia [18] proposed a model for investigating the stick - slip motion in a 2-DOF oscillator under arbitrary excitation, and provided a numerical approach to investigate such system. In the next year, Pilipchuk and Tan [19] studied the friction-induced vibration of a 2-DOF mass-damper-spring system interacting with a decelerating rigid strip.

Impact and friction usually occur at the same time in mechanical engineering. In 1994, Chin et al. [20] used Nordmark map to study several kinds of grazing bifurcations in a periodically forced impact oscillator with extra friction. Bapat [21] studied the *N*-impact-per-cycle periodic motions in an inclined impact damper with friction by theoretical predictions and numerical simulations in 1995. In the same year, Cone and Zadoks [22] investigated periodic motions and the corresponding stability and bifurcations of an impact oscillator with dry friction by a numerical method. Hinrichs et al. [23] discussed the nonlinear dynamics of oscillators with impact and friction in 1997. In 1999, Virgin and Begley [24] used the interpolated cell mapping method to determine the grazing bifurcation and attraction basin of an impact-friction oscillator. In 2000, Blazejczyk-Okolewska [25] investigated the bifurcation diagrams of an impact oscillator with external periodic forces and dry friction by numerical analysis. In 2002, Andreaus and Casini [26] gave closed-form solutions of a Coulomb friction-impact model without external excitation. In 2014, Burns and Piiroinen [27] studied the complexity of a basic impact mapping for rigid bodies with impact and friction. In 2015 and 2016, Bazhenov et al. [28,29] gave the stability and bifurcation analysis for 2-DOF vibro-impact system by parameter continuation method, and numerical bifurcation analysis of discontinuous 2-DOF vibro-impact system, respectively.

At present, the complexity of dynamical behaviors in friction or impact oscillators are also difficult to be investigated. In recent years, the theories of the discontinuous dynamical systems on time-varying domains and boundaries were gradually formed. In 2005, Luo [30] developed a theory to investigate the non-smooth dynamical systems on connectable and accessible subdomains. In 2008, Luo [31] introduced the G-functions for discontinuous dynamical systems to investigate the switchability of flows on discontinuous boundaries. Luo [32] developed the flow switchability theory of discontinuous dynamical systems on time-varying domains, and introduced different systems to model practical problems by this theory in 2009. In the same year, Luo et al. [33–36] investigated mechanism of impacting chatter with stick, periodic motions and chaos with impacting chatter or stick in a gear transmission system, periodic motion in a simplified brake system with a periodic excitation, and analytical conditions for motion switchability in a 2-DOF friction-induced oscillator moving on two constant speed belts, respectively. In 2011, Guo and Luo [37] systematically investigated the switching bifurcation and chaos in a periodically excited horizontal impact pair. In the next year, the related theory of discontinuous dynamical systems is given systematically by Luo [38], such as singularity and flow passability, flow barriers and switchability, transport laws and multi-valued vector fields, switchability and attractivity of domain flows, dynamics and singularity of boundary flows, edge dynamics and switching complexity etc. Using the flow switchability theory of discontinuous dynamical systems, some results of discontinuous dynamics in some classical mechanical models have been obtained, such as Refs [8–10,32–37,39–46,48]. In 2013 and 2014, Luo et al. [39,40] studied singularity, switchability and bifurcations in a 2-DOF periodically forced, frictional oscillator, and discontinuous dynamics of a freight train suspension system, respectively. Chen and Fan [41] developed the analytical conditions for non-stick motions and grazing motions of a double-belt friction oscillator in 2016. In 2018, Fan et al. [42] further studied the dynamical behaviors of the extended double-belt

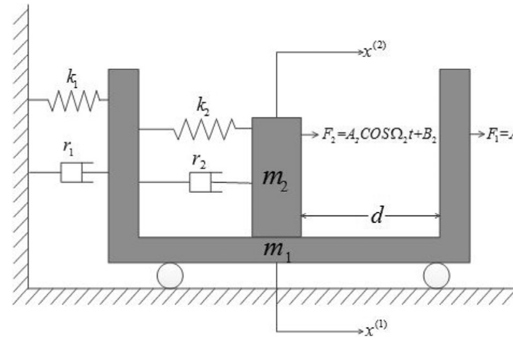

Fig. 1. Physical model.

friction oscillator modelled from linear vibration screen. Li et al. [43] studied passable motions and stick motions of a friction-induced oscillator with 2-DOF on a speed-varying belt in 2016, and Fan et al. [44] further investigated the dynamical behaviors for such an oscillator in 2017. In 2017 and 2018, Fan et al. [45–47] investigated the dynamical behaviors of a friction-induced oscillator with a straight line switching law or an elliptic control law or with impact. Zhang and Fu [48] developed mechanical complexity in a horizontal impact pair with dry friction in 2017. Other discontinuous models with delay or impulse or Boolean control have also been investigated extensively, for example [49–65].

In this paper, the discontinuous dynamics in a 2-DOF vibro-impact system with dry friction is investigated by using the flow switchability theory for discontinuous dynamical systems. The paper is outlined as follows. In Section 2, the physical model of a 2-DOF vibro-impact oscillator with friction are introduced. Furthermore, the motions of the oscillator are divided into four cases of non-stick motion, sliding-stick motion, side-stick motion and impacting motion; and the corresponding motion equations are also given. In Section 3, the partitions of phase plane and the discontinuous boundaries of motion in such oscillator are discussed in absolute coordinates and relative coordinates, respectively. In Section 4, the fundamental theory on flow switchability of discontinuous dynamical systems is presented; and the analytical conditions for the motion switching on discontinuous boundaries are developed. The switching sets and mapping structures are introduced through discontinuous boundaries in Section 5. In Section 6, the numerical simulations of sliding-stick motion, side-stick motion and grazing motion are given to illustrate the analytical conditions of motion switching in this discontinuous system. Finally, Section 7 concludes the paper.

In the previous study of dynamical behavior of this model, only numerical analyses were carried out for determining the dynamic responses of the vibro-impact system with dry friction, and the flow switching on discontinuous boundaries wasn't discussed. In this paper, the analytical conditions of flow switching on all possible discontinuous boundaries will be given, and the corresponding numerical simulations of passable motion, sliding-stick motion, side-stick motion and grazing motion will be carried out. Furthermore, the analytical prediction of periodic motions will be carried out by defining switching sets and constructing basic or four-dimensional mappings. This investigation on the impact oscillator with dry friction has important significance in the optimization design of machinery with clearance, noise suppression and reliability.

2. Physical model

A 2-DOF vibro-impact system with dry friction consists of two masses, which is shown in Fig. 1. The primary mass m_2 is connected to the secondary mass m_1 via a spring of stiffness k_2 and a damper of viscous damping coefficient r_2. The secondary mass