Chaos, Solitons and Fractals 116 (2018) 43–53

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Fractional hyper-chaotic model with no equilibrium

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ARTICLE INFO

Article history:
Received 13 August 2018
Revised 8 September 2018
Accepted 8 September 2018

Keywords:
Fractional hyper-chaotic model
Atangana–Baleanu derivatives
Numerical scheme
Fundamental theorem of fractional calculus

ABSTRACT

This paper considers a novel four dimensional dynamical model containing hyper-chaotic attractors. The concept fractional differential based on the exponential and Mittage–Leffler kernel were used to extend the classical version in order to include into the mathematical formulation the crossover in waiting time distribution. We have presented for both models the conditions under which the existence and the uniqueness of exact solutions are reached.We have used a newly established numerical scheme, that combines the fundamental theorem of fractional calculus and the Lagrange interpolation polynomial to solve the system numerically. We presented some numerical simulations for different values of fractional order, we compared both models with the existing one under the framework of fractional calculus. Our model has shown very new chaotic features in particular with the Atangana–Baleanu fractional derivative.

1. Introduction

Chaos has revalorized the development of technology as soon as its appearance in 1963. It is interesting that chaotic system have large range of application in several Engineering and non-engineering fields [1–3]. Furthermore Lorenz discovered chaotic attractors. After that Rossler [6] continued this work and proposed a new chaotic system. It is now well established from Wang [7] found a hyper chaotic system without equilibrium. Recent studies have attempted to explore special features of no equilibrium hyper chaotic system. Doungmo and Nieto [16] described some fractional models and numerical approximations for some attractor points. A system modeling evolutionary language and learning dynamics is investigated using the crank- Nicholson numerical method. Some of its convergence condition are in details see [18]. The study of the behaviors of replicator- mutator in which Caputo–Fabrizio operator is used in [17] presented a concrete application to replicator- mutator dynamics for a population.

The aim of this paper to introduce a new fractional chaotic model with no equilibrium thus giving a positive answer to above proposed question. A four dimensional chaotic system consists of four multipliers terms and four simple terms. Compared with all the proposed chaotic system until now, the biggest difference and most attractive place that there exists no equilibrium point in this system. The fractional chaotic system means of the improved version of classical system in which we apply Atangana–Toufik numerical algorithm and the chaotic attractor can be observed for suitable order [4]. In this paper, we extend the model under investigation by employing the Atangana–Baleanu derivative [5]. We construct a new numerical scheme for this model with Atangana–Baleanu derivative with non local and non-singular kernel then we apply fundamental theorem of fractional calculus and using Lagrange's polynomial interpolation. Readers can find the following interesting literatures about fractional differntial operators with non-singular and non-local kernel and also some applications of fractional differential operatos to modeling chaotic problems [8–15]

The organization of paper is as follows. Some basic definitions and properties of fractional calculus are short in Section 2. The stability and uniqueness condition are derived in Section 3. Presents Numerical solution via Atangana–Toufik numerical scheme in Section 4. Comparing the existence model in the fractional case Section 5. Some numerical experiments are given in Section 6. We conclude the paper with Section 6.

2. Numerical scheme with Caputo fractional derivative

In this section we propose a novel 4D character system which is hyperchaotic system which is hyper chaotic with no equilibrium. This multi character system is defined as follows

$$\begin{cases} x'(t) = -ay^2(t) \\ y'(t) = bw(t)z(t) + d \\ z'(t) = y^4(t) - cz^2(t) + c \\ w'(t) = x^{1.5}(t) + y^2(t) - w(t)z(t) - y^2(t)z(t) \end{cases} \tag{1}$$

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https://doi.org/10.1016/j.chaos.2018.09.009

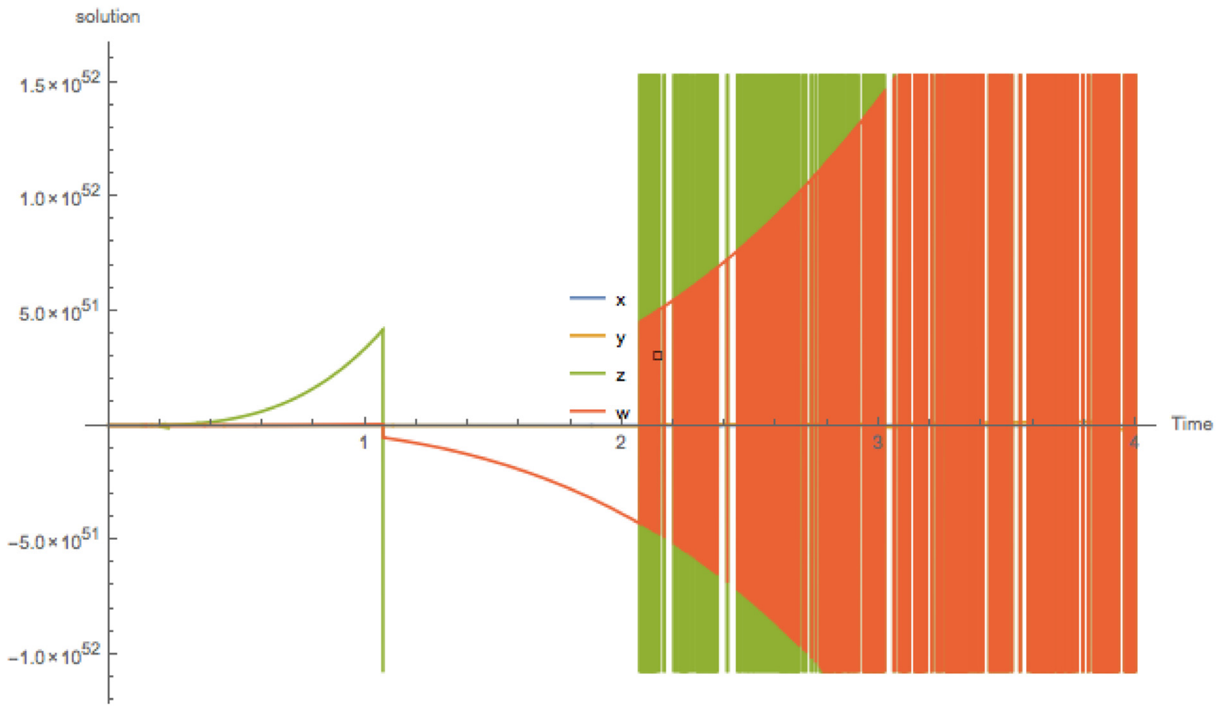

Fig. 1. Numerical simulation of the model with ABC for alpha = 0.99.

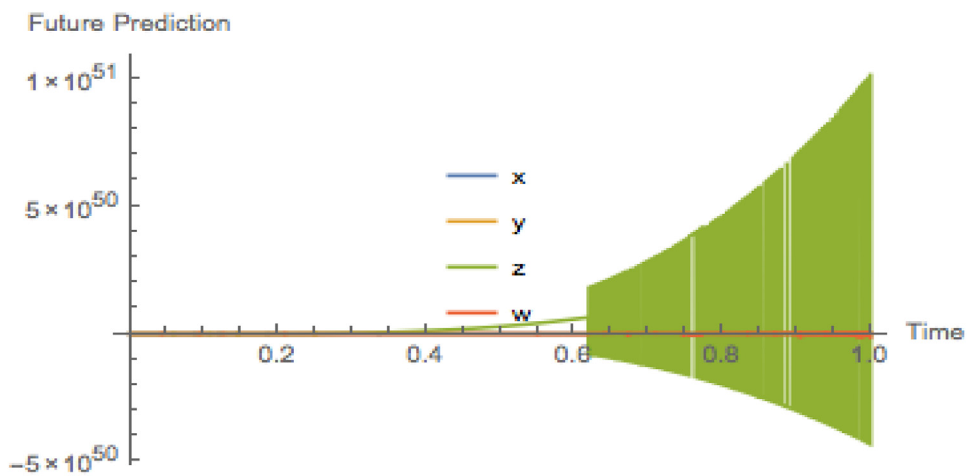

Fig. 2. Numerical simulation with ABC for alpha = 0. 85.

To apply Atangana–Toufik method [5] we convert the above to

$$\begin{cases} x = f_1(x, y, z, t) \\ y = f_2(x, y, z, t) \\ z = f_3(x, y, z, t) \\ w = f_4(x, y, z, t) \end{cases} \tag{2}$$

To solve the above we transform to

$$\begin{cases} x(t) - x_0 = \int_0^t f_1(x, y, z, \tau)d\tau \\ y(t) - y_0 = \int_0^t f_2(x, y, z, \tau)d\tau \\ z(t) - z_0 = \int_0^t f_3(x, y, z, \tau)d\tau \\ w(t) - w_0 = \int_0^t f_4(x, y, z, \tau)d\tau \end{cases} \tag{3}$$

With Caputo fractional derivative we have
The above can be converted to

$$x(t) - x_0 = \frac{1}{\Gamma(\alpha)} \int_0^t f_1(x, y, z, \tau)\,(x - \tau)^{\alpha-1} d\tau \tag{4}$$

$$y(t) - y_0 = \frac{1}{\Gamma(\alpha)} \int_0^t f_2(x, y, z, \tau)(x - \tau)^{\alpha-1} d\tau \tag{5}$$

$$z(t) - z_0 = \frac{1}{\Gamma(\alpha)} \int_0^t f_3(x, y, z, \tau)(x - \tau)^{\alpha-1} d\tau \tag{6}$$

$$w(t) - w_0 = \frac{1}{\Gamma(\alpha)} \int_0^t f_4(x, y, z, \tau)(x - \tau)^{\alpha-1} d\tau \tag{7}$$

By applying the fundamental theorem of fractional calculus, The above equations can be converted to fractional integral equation

$$x(t) - x(0) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f_1(x, y, z, \tau) d\tau \tag{8}$$

At the point $t_{n+1,\ n=0,1,2,.........}$ So the above equation is of the form as [5]

$$x(t_{n+1}) - x(0) = \frac{1}{\Gamma(\alpha)} \int_0^{t_{n+1}} (t_{n+1} - \tau)^{\alpha-1} f_1(x(\tau), y(\tau), z(\tau), w(\tau))\, d\tau \tag{9}$$