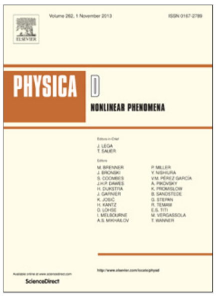

Synchronisation vs. resonance: Isolated resonances in damped nonlinear oscillators

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PII: S0167-2789(17)30286-5
DOI: https://doi.org/10.1016/j.physd.2018.05.004
Reference: PHYSD 32024

To appear in: *Physica D*

Received date : 22 May 2017
Revised date : 16 February 2018
Accepted date : 25 May 2018

Please cite this article as: A. Marchionne, P. Ditlevsen, S. Wieczorek, Synchronisation vs. resonance: Isolated resonances in damped nonlinear oscillators, *Physica D* (2018), https://doi.org/10.1016/j.physd.2018.05.004

Synchronisation vs. Resonance: Isolated Resonances in Damped Nonlinear Oscillators

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(Dated: May 30, 2018)

We describe differences between synchronisation and resonance, and analyse different types of nonlinear resonances in a weakly damped Duffing oscillator using bifurcation theory techniques. In addition to previously reported (i) odd subharmonic resonances found on the primary branch of symmetric periodic solutions with the forcing frequency and (ii) even subharmonic resonances due to symmetry-broken periodic solutions that bifurcate off the primary branch and also oscillate at the forcing frequency, we uncover (iii) novel resonance type due to isolas of periodic solutions that are not connected to the primary branch. These occur between odd and even resonances, oscillate at a fraction of the forcing frequency, and give rise to a complicated resonance 'curve' with disconnected elements and high degree of multistability.

We use bifurcation continuation to compute resonance tongues in the plane of the forcing frequency vs. the forcing amplitude for different but fixed values of the damping rate. Our analysis shows that identified here isolated resonances explain the intriguing "intermingled tongues" that were observed for weak damping and misinterpreted as (synchronisation) Arnold tongues in [Phys. Rev. E 57, 1554 (1998)]. What is more, isolated resonances link "intermingled tongues" to a seemingly unrelated phenomenon of "bifurcation superstructure" described for moderate damping in [Phys. Lett. A 107, 351 (1985)].

INTRODUCTION

Many complex systems in the natural world and technology show oscillatory behaviour, either as self-sustained oscillations, or as response to external forcing. Even though the description of the detailed dynamics is often incomplete, such systems can be modelled and understood in terms of low-dimensional nonlinear oscillators that capture the dominating degrees of freedom. Throughout the paper, we refer to two oscillator types:

- *damped oscillators*: linear or nonlinear dissipative dynamical systems that exhibit oscillations whose amplitude decays to zero over time (e.g. stable equilibrium with a pair of complex conjugate eigenvalues), and
- *dissipative self-sustained oscillators*: nonlinear dissipative dynamical systems that exhibit self-sustained oscillations (e.g. a stable limit cycle).

When an oscillator is subject to external periodic forcing, two widely studied phenomena are at play, depending on the behaviour of the unforced system. Firstly, a linear (harmonic) damped oscillator exhibits increased amplitude of oscillations when forced near its natural frequency. The situation becomes surprisingly more complex in ubiquitous nonlinear damped oscillators, which exhibit increased amplitude of oscillations together with bistability (or even multistability) near several subhamonic forcing frequencies. This is the phenomenon of resonance [1, 2]. Secondly, when dissipative self-sustained oscillators are subject to external periodic forcing, or coupled to one another, they may lock their frequencies at different ratios. This is the phenomenon of synchronisation, which is even more complicated than nonlinear resonances [2].

Resonance and synchronisation phenomena are observed in a variety of natural systems, ranging from biology [3, 4] to glacial cycles [5, 6]. Also, various physical systems, such as lasers [7, 8], electronic circuits [9] and mechanical pendulums [10] have been studied in this framework. Though externally forced and coupled oscillators have been studied extensively throughout the last century, some confusion between resonance and synchronisation exists in the literature [11].

Identifying the characteristic properties as well as differences between the phenomenon of resonance and that of synchronisation has several advantages to studying nonlinear dynamics of oscillating complex systems. It can be a valuable tool in construction of simplified models that capture the essential nonlinearities and are more amenable to analysis, thus allowing for an understanding of the underlying physical mechanisms. It can also provide valuable guidance in the analysis of complex systems (e.g. climate) where the detailed dynamics is often unknown and must be inferred from observations. Here, it can help enlighten the underlying mechanisms behind, say, a change in the observed oscillation frequency without any change in the forcing. In situations where the system cannot be separated from the forcing (e.g. climate paced by astronomical forcing) it can help answer questions about the intrinsic dynamics of the unforced system and the origin of oscillations. What is more, there are systems (e.g. lasers) where damped oscillations occur on top of self-sustained oscillations (e.g. stable limit