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REVIEW ARTICLE

Pair copula constructions to determine the dependence structure of Treasury bond yields

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Abstract We estimated the dependence structure of US Treasury bonds through a pair copula construction. As a result, we verified that the variability of the yields decreases with a longer time of maturity of the bond. The yields presented strong dependence with past values, strongly positive bivariate associations between the daily variations, and prevalence of the Student's t copula in the relationships between the bonds. Furthermore, in tail associations, we identified relevant values in most of the relationships, which highlights the importance of risk management in the context of bonds diversification.

Introduction

Since the introduction of the mathematical theory of portfolio selection and of the capital asset pricing model (CAPM), the issue of dependence has always been of fundamental importance to financial economics. In the context of international diversification, there is a need to minimise the risk of specific assets (such as stocks and Treasury bonds) through optimal allocation of resources. Many studies have used a statistical model which is able to measure the temporal dependence between stocks and Treasury bonds: Campbell and Ammer (1993) apply a vector autoregressive (VAR) system in AMEX and NYSE stocks and US Treasury bonds, but they do not analyse the effect of the volatility of the relationship. Li (2002) and Kim, Moshirian, and Wu (2006) estimate a bivariate generalised autoregressive conditional heteroscedasticity (GARCH) model and bivariate exponential GARCH with t-distribution and verify important implications in stock-bonds correlation. However, Cappiello, Engle, and Sheppard (2006), and Li and Zou (2008) expand the asymmetric and multivariate approach with dynamic conditional correlation (DCC) GARCH.

Traditionally, correlation is used to describe the dependence between random variables, but recent studies, such as that conducted by Embrechts, Lindskog, and McNeil (2003), have ascertained the superiority of copulas to model dependence. Copulas offer much more flexibility than the correlation

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approach because a copula function can deal with non-linearity, asymmetry, serial dependence and also the well-known heavy-tails of financial assets' marginal and joint probability distribution. In studies of Treasury bonds, Junker, Szimayer, and Wagner (2006) apply the normal copula model in US Treasury monthly bonds, confirming the importance of this approach in considering tail dependence and symmetry. Lee, Kim, and Kim (2011) apply Archimedean copulas in interdependence of US, UK, and Japan interest rates, according to different maturities of bonds. This paper verifies that both negative and positive returns in the US and UK move in a similar trend whereas in Japan interest rates follow a different trend. Diks et al. (2014) test the forecast accuracy of copula families in 10-years maturity of G7 countries' government bonds, where the Student's t and Clayton mixture copula outperforms the other copulas considered.

A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector, which is the copula. Although the literature on copulas is consistent, the great part of the research is still limited to the bivariate case. Thus, constructing higher dimensional copulas is the natural next step, but this is not an easy task. Apart from the multivariate Gaussian and Student (see work in stock-bonds structure dependence of Kang, 2007), the selection of higher-dimensional parametric copulas is still rather limited (Genest, Rémillard, & Beaudoin, 2009).

The developments in this area tend to hierarchical, copula-based structures. It is possible that the most promising of these is the pair copula construction (PCC). Originally proposed by Joe (1996), it has been further discussed and explored in the literature for questions of inference and simulation. The PCC is based on a decomposition of a multivariate density into bivariate copula densities, of which some are dependency structures of unconditional bivariate distributions, and the rest are dependency structures of conditional bivariate distributions. Applications to financial data have shown that these vine-PCC models outperform other multivariate copula models in predicting log-returns of equity portfolios. Min and Czado (2010) present a PCC copula model in daily returns from January 1, 1999 to July 8, 2003 of the Norwegian stock index, the MSCI world stock index, the Norwegian bond index and the SSBWG hedged bond index, and they verify a stronger dependence between international bonds and stocks, international and Norwegian stocks, and Norwegian stocks and bonds, but they observe that the Norwegian bond index does not depend on the MSCI world stock index if the Norwegian stock index is given. In this context, this paper poses the question: What would the dependence structure of Treasury bonds be in relation to their maturity?

To answer this question, this paper aims to estimate the dependence structure between Treasury bonds through a PCC. To that effect, we collected daily data from Treasury bonds of the US government for 1-, 2-, 3-, 5-, 7- and 10-years of maturity, which were the most sought after by investors in order to obtain truly risk free assets. The estimated structure allows the calculation of the non-linear absolute and tail dependences of each bivariate relationship between the bonds, isolating the effect of the other. It is also possible to verify which bond has more dependence with all the others, and to identify the "leading" Treasury bonds.

The paper is structured as follows: The second section briefly presents the background of copulas and PCC; the third section presents the material and methods of the study, describing the data and the procedures used to achieve the objective of the paper; the fourth section presents the results obtained and the discussion; and the fifth section contains the conclusions of the paper; the appendix introduces the copula families utilised in this study.

Background

This section is subdivided into: i) Copula methods, which briefly defines this class of functions and describes its properties; this sub section also contains a literature review; ii) Pair copula construction, which succinctly describes the concepts of this structure.

Copula methods

Dependence between random variables can be modelled by copulas. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as the univariate marginal behaviour of random variables can be modelled separately from their dependence (Kojadinovic & Yan, 2010).

The concept of copula was introduced by Sklar (1959). However, it was only recently that its applications became clear. A detailed treatment of copulas as well as of their relationship to concepts of dependence is given by Joe (1997) and Nelsen (2006). A review of the applications of copulas to finance can be found in Embrechts et al. (2003) and in Cherubini, Luciano, and Vecchiato (2004).

To facilitate our understanding of the concept we restrict our attention to the bivariate case. The extensions to the *n*-dimensional case are straightforward. A function $C:[0,1]^2 \to [0,1]$ is a copula if, for $0 \le x \le 1$ and $x_1 \le x_2, y_1 \le y_2, (x_1, y_1), (x_2, y_2) \in [0,1]^2$, it fulfills the following properties:

$$C(x,1) = C(1,x) = x, \quad C(x,0) = C(0,x) = 0. \tag{1}$$

$$C(x_2, y_2) - C(x_2, y_1) - C(x_1, y_2) + C(x_1, y_1) \ge 0. \tag{2}$$

Property (1) means uniformity of the margins, while (2), the n-increasing property means that $P(x_1 \le X \le x_2, y_1 \le Y \le y_2) \ge 0$ for (X, Y) with distribution function C.

In Sklar's seminal paper (1959), it was demonstrated that a copula is linked with a distribution function and its marginal distributions. This important theorem states:

(i) Let C be a copula and F_1 and F_2 univariate distribution functions. Then (3) defines a distribution function F with marginals F_1 and F_2.

$$F(x,y) = C(F_1(x), F_2(y)), (x,y) \in R^2. \tag{3}$$

(ii) For a two-dimensional distribution function F with marginal F_1 and F_2, there is a copula C satisfying (3). This is unique if F_1 and F_2 are continuous and then, for every