Journal of Business Research 68 (2015) 1574–1580

Contents lists available at ScienceDirect

Journal of Business Research

Aggregation methods to calculate the average price ☆

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ARTICLE INFO

Available online 16 March 2015

Keywords:
Average price
Aggregation systems
Unified aggregation operator
OWA operator

ABSTRACT

Average price is a numerical value that represents a set of prices, which may relate to firms, countries, or regions. This study presents new methods of average price aggregation that build on the unified aggregation operator (UAO). The UAO combines a wide range of sub-aggregation processes into a single formulation capable of accounting for the importance of each concept in the analysis. The aggregation system is flexible, can adapt to different environments, and provides a complete representation of relevant information. The UAO can calculate the average price for numerous geographical contexts such as supranational regions and countries. The study illustrates the UAO's utility by presenting an example of how to calculate the world average price of a product while considering a range of opinions and environmental uncertainties.

1. Introduction

Average price is a representative value for a set of varying prices (Chen, 2006; Silver & Ioannidis, 2001). Average prices are useful to analyze prices within sets of firms, countries, or regions and are a fundamental part of pricing research (Johnson & Cui, 2013; Leone, Robinson, Bragge, & Somervuori, 2012). Usually, analysis of the average price uses the simple average or the weighted average. However, many other aggregation techniques exist (Beliakov, Pradera, & Calvo, 2007; Grabisch, Marichal, Mesiar, & Pap, 2011). An increasingly popular aggregation operator is the ordered weighted average (OWA) (Yager, 1988; Yager, Kacprzyk, & Beliakov, 2011). The OWA is an aggregation operator that provides a parameterized family of aggregation operators that range between the minimum and the maximum. The main advantage is its efficiency in representing the decision maker's attitudinal character.

Recent studies employ, several generalizations of the OWA operator and related techniques, which include the integration of weighted averages (Torra, 1997; Xu & Da, 2003) and probabilities with the OWA operator (Engemann, Filev, & Yager, 1996; Yager, Engemann, & Filev, 1995). Merigó (2012) introduces the probabilistic OWA operator and further generalizations that include the weighted average (Merigó, Lobato-Carral, & Carrilero-Castillo, 2012). The main advantage of this approach is the flexibility to be able to consider different information sources in the same formulation. An additional practical development is the unified aggregation operator (UAO) (Merigó, 2011). The UAO has a more general structure and is capable of additional aggregation that can capture numerous sub-aggregation processes within the specific problem while accounting for each concept's importance.

This study sets forth new methods for calculating the average price by using the UAO and related techniques. This approach provides a more general representation of the information by considering different information sources such as the experts' opinion, probabilities, weighted averages, and the decision maker's attitudinal character. In addition, the UAO draws on information from firms, countries, and regions. The UAO's main advantage is its flexibility in adapting to the specific needs of the environment.

This study addresses key examples including the average price in the European Union, North America, and Asia. This approach lets decision makers analyze different information types and integrate these information types into a representative result that fits the decision maker's interests. The study presents an example for the world average price to illustrate the method numerically. To demonstrate the UAO's capacity to include sub-structures in average price calculation, the study also analyzes the US average price and the average price of the individual US states.

Section 2 briefly reviews some key aggregation systems. Section 3 introduces new methods for addressing the average prices. Section 4 presents an illustrative example. Finally, Section 5 summarizes the study's main findings.

☆ The authors would like to thank Professor Domingo Ribeiro, University of Valencia, Professor Jian-Bo Yang, University of Manchester, Professor Pedro Soto, University of Murcia, Professor Shouzhen Zeng, Zhejiang Wanli University, the guest editors of this special issue, the anonymous reviewers and the participants of the GIKA 2014 conference for their valuable comments, which have improved the quality of this research. The authors also gratefully acknowledge the support from the European Commission (PIEF-GA-2011-300062) and the Spanish Ministry of Economy and Competitiveness (DER2012-39223-C02-02).

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http://dx.doi.org/10.1016/j.jbusres.2015.01.054
0148-2963/

2. Aggregation systems

Aggregation systems are very common in the literature (Beliakov et al., 2007; Grabisch et al., 2011). They capture initial information, summarize results, and give decision makers a better representation of the available information. The simple average, the weighted average and the ordered weighted average (OWA) (Yager, 1988) are the most popular aggregation systems. Assessing complex information, however, requires methods that are more general. A useful tool for assessing complex information is the unified aggregation operator (UAO) (Merigó, 2011). The definition of the UAO is as follows.

Definition 1. A unified aggregation operator of dimension m is a mapping UAO: $R^m \times R^n \rightarrow R$, with an associated weighting vector C of dimension m representing concepts with a degree of importance C_h, such that

$$UAO(a_1, \ldots, a_n) = \sum_{h=1}^{m}\sum_{i=1}^{n} C_h w_i^h a_i, \tag{1}$$

where C_h is the degree of importance of each concept in the aggregation such that $C_h \in [0, 1]$ and $\sum_{h=1}^{m} C_h = 1$; and w_i^h is the ith weight of the hth weighting vector W such that $w_i^h \in [0, 1]$ and $\sum_{i=1}^{n} w_i^h = 1$.

The UAO includes a wide range of aggregation operators including the weighted average, the OWA operator and the POWAWA operator (Merigó et al., 2012). Note that if some of the aggregation methods do not appear in the order according to i, the reordering of these sub-aggregation operators to the ordering of i is necessary.

The UAO contains the POWAWA operator when using probability, the weighted average and the OWA operator. Thus,

$$f(a_1, \ldots, a_n) = C_1 \sum_{j=1}^{n} w_j b_j + C_2 \sum_{i=1}^{n} v_i a_i + C_3 \sum_{i=1}^{n} p_i a_i. \tag{2}$$

According to Eq. (1), conversion of w_j and b_j is possible using x_i and a_i, where x_i is the ith weight w_j ordered according to the initial positions i. This formulation, automatically yields other basic cases as follows:

- If $C_1 = 1$, the formulation yields the OWA operator.
- If $C_2 = 1$, the formulation yields the weighted average.
- If $C_3 = 1$, the formulation yields the probabilistic aggregation.
- If $C_1 = 0$, the formulation yields the PWA operator.
- If $C_2 = 0$, the formulation yields the POWA operator.
- If $C_3 = 0$, the formulation yields the OWAWA operator.

Note that the UAO includes many other types of aggregation operators. The idea is to use the specific cases pertinent to the problem at hand. Other popular aggregation operators are the Choquet integrals (Belles, Merigó, Guillen, & Santolino, 2014), distance measures (Zeng, Su, & Le, 2012), generalized aggregation operators (Zhou, Chen, & Liu, 2013) and fuzzy systems (Zhao, Lin, & Wei, 2013).

3. Analysis of average prices

Many methods analyze and calculate average prices (Silver & Ioannidis, 2001). This research presents new methods of average price calculation that can represent the information more completely. These approaches must be flexible to adapt to the specific needs of the problem. The UAO offers this flexibility. The UAO generalizes a range of aggregation operators. Using the UAO in the simplest representation of the average price gives the following expression:

$$P = \sum_{h=1}^{m}\sum_{i=1}^{n} C_h w_i^h p_i, \tag{3}$$

where p_i is the price of the ith firm or region; w_i^h is the ith weight of the hth weighting vector W such that $w_i^h \in [0, 1]$ and $\sum_{i=1}^{n} w_i^h = 1$; and C_h is the firm's or region's importance in the aggregation such that $C_h \in [0, 1]$ and $\sum_{h=1}^{m} C_h = 1$.

This formulation considers only one set of elements with specific prices. In the real world, however, more sets are often available, and problems must sometimes take into account firms, regions, and countries. Such cases require a more general structure. Manipulating Eq. (3) yields the following:

$$P = \sum_{h=1}^{m}\sum_{g=1}^{o}\sum_{i=1}^{n} C_h v_{hg} w_i^{hg} p_{ihg}, \tag{4}$$

where p_{ihg} is the price for the gth country in the ith region (or firm) and hth opinion; w_i^{hg} is the weight (or market share) of the ith region in the gth country such that $w_{ig} \in [0, 1]$ and $\sum_{i=1}^{n} w_{ig} = 1$.

Observe that many examples could follow this direction. For example, under the assumption that three weighting vectors represent the objective and subjective information and the attitudinal character, the POWAWA operator would be suitable for analysis. Here, Eq. (4) could take the following form:

$$P = C_1 P_{OWA} + C_2 P_{WA} + C_3 P_{PA} \tag{5}$$

or:

$$P = C_1 \sum_{g=1}^{o}\sum_{i=1}^{n} v_{1g} w_i^{1g} p_{i1g} + C_2 \sum_{g=1}^{o}\sum_{i=1}^{n} v_{2g} w_i^{2g} p_{i2g} + +C_3 \sum_{g=1}^{o}\sum_{i=1}^{n} v_{3g} w_i^{3g} p_{i3g}. \tag{6}$$

Scholars could study many other variations of this approach following the UAO approach (Merigó, 2011).

The next part of the discussion explores some interesting real world examples. To develop these examples, the first step is to present the formulas for calculating the average price in some representative supranational regions. Table 1 displays the results.

In this example, each country comprises several sub-aggregations (i.e., states, provinces, and firms). Table 2 displays such a structure for the USA.

The order of each formula runs from the highest economy to the lowest economy. Standard definitions from well-known sources define the regions. Some small differences may appear, however, because of the specific conditions in each country. Note that the real world is more complex because of the need to consider firms, scenarios, and differences between cities and towns.

A major advantage of using this approach is that the analysis does not lose information because the minimum and the maximum bound the results:

$$\text{Min}\{p_i\} \leq \text{AP} \leq \text{Max}\{p_i\}, \tag{7}$$

where AP is the average price that an aggregation system (e.g., simple average of the UAO) yields.

Observe that the UAO admits numerous partial bounds that account for the minimum and the maximum when considering additional information. A representation of this idea is as follows:

$$\text{Min}\{p_i\} \leq \ldots \leq \text{Min-UAO}\{p_i\} \leq \text{AP} \leq \text{Max-UAO}\{p_i\} \leq \ldots \leq \text{Max}\{p_i\}, \tag{8}$$

where Min-UAO and Max-UAO indicate an aggregation process that combines the minimum and the maximum with the UAO, and Min-UAO and Max-UAO are the minimum and maximum of the additional aggregation integrated within the UAO.

Finally, this methodology closely resembles box-plot analysis (Tukey, 1977) and other statistical methods. The difference in the UAO approach is that the weighting vectors represent a specific attitude against the uncertainty of the environment. Moreover, the sub-