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Local well-posedness of the three dimensional compressible Euler–Poisson equations with physical vacuum

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ABSTRACT

This paper is concerned with the three dimensional compressible Euler–Poisson equations with moving physical vacuum boundary condition. This fluid system is usually used to describe the motion of a self-gravitating inviscid gaseous star. The local existence of classical solutions for initial data in certain weighted Sobolev spaces is established in the case that the adiabatic index satisfies $1 < \gamma < 3$.

RÉSUMÉ

Dans cet article on considère les équations d'Euler–Poisson tridimensionnelles compressibles avec conditions aux bords correpondant à un vide physique en mouvement. Ce système de fluide sert souvent à décrire le mouvement d'une étoile gazeuse non visqueuse avec un champ gravitationnel. On établit l'existence locale des solutions classiques dans certains espaces de Sobolev, pour des problèmes avec données initiales, dans le cas où l'indice adiabatique satisfait $1 < \gamma < 3$.

1. Introduction

1.1. Model and physical vacuum

The motion of a self-gravitating inviscid gaseous star in the universe can be described by the following free boundary problem for the compressible Euler equations coupled with the Poisson equation:

$$\rho_t + \nabla_\eta \cdot (\rho u) = 0 \qquad \text{in } \ \Omega(t), \tag{1.1a}$$

$$\rho[u_t + u \cdot \nabla_\eta u] + \nabla P = \rho \nabla_\eta \phi \qquad \text{in } \ \Omega(t), \tag{1.1b}$$

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$$-\Delta_{\eta\eta}\phi = 4\pi g\rho \qquad \text{in} \ \ \Omega(t), \tag{1.1c}$$

$$\nu(\Gamma(t)) = u \cdot n(t) \qquad \text{on} \ \ \Gamma(t), \tag{1.1d}$$

$$(\rho, u) = (\rho_0, u_0) \qquad \text{in} \ \ \Omega(0), \tag{1.1e}$$

$\eta = (\eta^1, \eta^2, \eta^3)$ denotes the spatial coordinate in $\mathbb{R}^3$. The open, bounded domain $\Omega(t) \subset \mathbb{R}^3$ denotes the changing domain occupied by the gas. $\Gamma(t) := \partial\Omega(t)$ denotes the moving vacuum boundary, $\nu(\Gamma(t))$ denotes the velocity of $\Gamma(t)$, and $n(t)$ denotes the exterior unit normal vector to $\Gamma(t)$. The density of gas $\rho(\eta, t) > 0$ in $\Omega(t)$ and $\rho = 0$ in $\mathbb{R}^3 \setminus \Omega(t)$. $u(\eta, t)$ denotes the Eulerian velocity field, $P(\eta, t)$ denotes the scalar pressure, $\phi(\eta, t)$ is the potential function of the self-gravitational force, and g is the gravitational constant. We consider a polytropic gas star, then the equation of state is given by:

$$P = C_\gamma \rho^\gamma \quad \text{for} \ \ \gamma > 1, \tag{1.2}$$

where C_γ is the adiabatic constant. We set both g and C_γ to be unity. We refer the readers to [3,10] for more details of the related background on this system.

The sound speed of equations (1.1) is given by $c := \sqrt{\partial P/\partial\rho}$, and N denotes the outward unit normal to the initial boundary $\Gamma := \partial\Omega(0)$, then the condition

$$-\infty < \frac{\partial c_0^2}{\partial N} < 0 \ \ \text{on} \quad \Gamma \tag{1.3}$$

defines a "physical vacuum" boundary, where $c_0(\cdot) = c(\cdot, 0)$. This definition of physical vacuum was motivated by the case of the Euler equations with damping studied in [36,39]. For more details and the physical background of this concept, please see [21,22,36,38,39,54].

The physical vacuum condition (1.3) is equivalent to the requirement that

$$\frac{\partial \rho_0^{\gamma-1}}{\partial N} < 0 \text{ on } \Gamma. \tag{1.4}$$

This condition is necessary for the gas particles on the boundary to accelerate. Since $\rho_0 > 0$ in Ω, (1.4) implies that for some positive constant C, when $x \in \Omega$ is close enough to the vacuum boundary Γ, then

$$\rho_0^{\gamma-1}(x) \geq C\text{dist}(x, \Gamma). \tag{1.5}$$

The physical vacuum boundary condition shows that the one order derivative of $\rho_0^{\gamma-1}$ has a jump on the vacuum boundary. This regularity is the main difficulty for establishing well-posedness theory for the free boundary problem. The condition (1.5) also describes density's decay behavior from inside domain to the vacuum interface. We can describe general decay behavior by

$$c_0^\alpha \geq C\text{dist}(x, \Gamma).$$

For $0 < \alpha \leq 1$, local well-posedness is proved for the compressible Euler equations with damping in [39]. In this case, $\rho_0^{\gamma-1}$ is smooth in the whole space, the degeneracy behavior is quite different from the physical vacuum case and the analysis for the case $0 < \alpha \leq 1$ is much simpler. For $\alpha > 2$ or $1 < \alpha < 2$, it is conjectured that the problem is ill-posedness. More details can be found in [22, Section V].

When the physical vacuum boundary condition is assumed, as we explained before, the compressible Euler equations become a degenerate and characteristic hyperbolic system, then the classical theory of hyperbolic systems cannot be directly applied. The local existence theory of classical solutions featuring the