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Mathematical problems in mechanics

Dynamic behavior of convergent rapid granular flows

Comportement dynamique d'un flux granulaire rapide convergent

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ABSTRACT

We consider a dilute flow of granular particles passing through a nozzle under gravity. This setting is an analogue to high-speed nozzle flows, which is a classical problem in the study of compressible gases. Contrary to the widely held belief that the behavior of very dilute granular systems is qualitatively similar to that of gases, we show that dilute granular systems can exhibit a type of intermittency that has no analogue in gas dynamics.

RÉSUMÉ

Nous considérons un flux dilué de particules granulaires passant par un tube d'injection sous l'action de la gravité. Ce problème est similaire à celui d'un écoulement de flux à grande vitesse, ce qui constitue un problème classique dans l'étude des gaz compressibles. Contrairement à l'idée très répandue que le comportement des systèmes granulaires très dilué est qualitativement semblable à celui des gaz, nous montrons que les systèmes dilués granulaires peuvent présenter un type d'intermittences qui n'a pas d'analogue dans la dynamique des gaz.

1. Introduction and formulation

Granular flows exhibit rich phenomena in their dynamic behaviors due to unique and complicated particle interactions. While the complication of dense granular systems has been attracting more attention, the behavior of dilute granular flows is under less investigation. Here we show that random collisional events between particles in dilute granular nozzle flows can lead to intermittency, the irregular alternation of phases in dynamical systems. This is of great interest in many applications where a steady flux of particles through the nozzle is required.

In this study, we will investigate the intermittent behavior of dilute granular flows. To be specific, we will study dilute flows of inelastic particles that are injected into a nozzle and fall under gravity g. To illustrate the mechanism in the simplest

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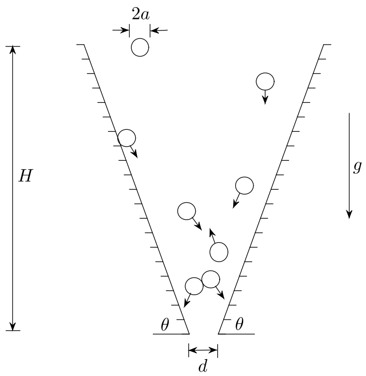

Fig. 1. Sketch of particles falling through a nozzle under gravity.

possible setting, we consider identical frictionless inelastic particles of radius a injected into a symmetric two-dimensional nozzle composed of rigid flat walls aligned at an angle θ to the horizontal. We denote the height of the portion of the nozzle above the bottleneck as H and the width of the bottleneck as d (see Fig. 1). The particles are injected from the top of the nozzle with zero velocity and injection frequency f. The locations of particle injection are randomly selected from a given distribution. We will focus on the simple case of a uniform distribution. We note that the fundamental mechanism for intermittency reported here also exists for much more general conditions.

In our system of dilute granular flows, particles travel relatively long distances between collisions and the principal mechanism for momentum transport between particles is via instantaneous collisional interactions. For simplicity, we will neglect frictional effects. Two different types of collisional interactions occur in this system: (1) collisions between a particle and a wall, i.e. particle–wall collisions, and (2) collisions between particles, i.e. interparticle collisions. Both types of collisions are dissipative. To quantify the energy losses inherent in these collisions, we introduce the coefficient of restitution e, which is defined as the ratio of relative speeds along the line of impact after and before a collision. Specifically, we have e_{w} for particle–wall collisions and e_{p} for interparticle collisions.

Each particle, between its entrance into the nozzle and its passage through the bottleneck, experiences a sequence of events, including instantaneous events and free-fall motion. Instantaneous events are the ones that occur exactly at a point of time t, including (1) entry into the nozzle, (2) collisions with the right wall, (3) collisions with the left wall, (4) collisions with other particles, and (5) exit through the bottleneck. Each event applies a change to the particle's velocity and/or location, which are denoted by $P^{(i)}(t) = (u^{(i)}(t), v^{(i)}(t), x^{(i)}(t), y^{(i)}(t))^{\mathrm{T}}$. Here $u^{(i)}(t)$ ($v^{(i)}(t)$) is the horizontal (vertical) velocity at time t, $x^{(i)}(t)$ ($y^{(i)}(t)$) is the horizontal (vertical) location at time t, and the superscript i is the index of the particle based on the order in which it was injected. For particle i, the time of the k-th collisional event occurring is denoted by $t_k^{(i)}$.

Each particle is released with zero velocity at the top of the nozzle, the width of which is $2D$ with $D = \frac{d}{2} + H\cot\theta$. We denote the time when particle i enters the system by $t_{\mathrm{in}}^{(i)}$, namely, $P_0^{(i)} = P^{(i)}(t_{\mathrm{in}}^{(i)}) = (0, 0, X^{(i)}, H)^{\mathrm{T}}$, where $X^{(i)}$ is a random variable under the probability density function

$$p(x) = \begin{cases} 1/(2D - 2a\csc\theta), & \text{when } |x| < D - a\csc\theta, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

When particle i hits the right nozzle wall, i.e. when $y^{(i)}/(x^{(i)} - d/2 + a\csc\theta) = \tan\theta$, its velocity component normal to the wall is reduced due to the inelastic collision, while the velocity component tangential to the wall remains unaffected. Therefore, its location and velocity are updated using the operator R defined by

$$P^{(i)}(t_k^{(i)+}) = RP^{(i)}(t_k^{(i)-}) = \begin{pmatrix} W(\theta) & 0_{2\times 2} \\ 0_{2\times 2} & I_{2\times 2} \end{pmatrix} P^{(i)}(t_k^{(i)-}), \tag{2}$$

where $I_{2\times 2}$ is a 2×2 identity matrix, $0_{2\times 2}$ is a 2×2 zero matrix, and

$$W(\theta) = \frac{1}{2}\begin{pmatrix} 2 - 2(1+e_{\mathrm{w}})\sin^2\theta & (1+e_{\mathrm{w}})\sin 2\theta \\ (1+e_{\mathrm{w}})\sin 2\theta & 2 - 2(1+e_{\mathrm{w}})\cos^2\theta \end{pmatrix}. \tag{3}$$

Similarly, when particle i collides with the left nozzle wall, i.e. when $y^{(i)}/(-x^{(i)} - d/2 + a\csc\theta) = \tan\theta$, its location and velocity are updated using the operator L defined by

$$P^{(i)}(t_k^{(i)+}) = LP^{(i)}(t_k^{(i)-}) = \begin{pmatrix} W(-\theta) & 0_{2\times 2} \\ 0_{2\times 2} & I_{2\times 2} \end{pmatrix} P^{(i)}(t_k^{(i)-}). \tag{4}$$

When particles i and j collide, i.e. when $(x^{(i)} - x^{(j)})^2 + (y^{(i)} - y^{(j)})^2 = 4a^2$, their velocities along the line of impact are determined by the conservation of momentum and the coefficient of restitution e_{p}, while the components of velocity