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An interface condition to compute compressible flows in variable cross section ducts

Conditions d'interface pour le calcul d'écoulements compressibles en conduite à section variable

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ABSTRACT

We propose an improved interface condition in order to account for head losses in pipe when some discontinuous cross sections occur.

RÉSUMÉ

Nous proposons d'améliorer la condition d'interface afin de prendre en compte la perte de charge pour un écoulement en conduite à section variable discontinue.

1. Introduction

We examine in this paper some possible improvement of the well-balanced strategy in order to compute approximations of solutions to Euler-type models in one-dimensional geometries corresponding basically to pipe configurations. From a practical point of view, these methods are very useful in order to build stable and efficient algorithms. If ρ, u, p respectively denote the density, the velocity and the pressure of the fluid, and noting as usual $\epsilon(p,\rho)$ the internal energy, $E = \rho\left(\epsilon(p,\rho) + u^2/2\right)$ the total energy, and $A(x)$ the – steady – cross section of the pipe, the target set of equations is

$$\begin{cases} \partial_t(A\rho) + \partial_x(A\rho u) = 0, \\ \partial_t(A\rho u) + \partial_x\left(A\rho u^2 + Ap\right) - p\,\partial_x A = A(x)S_u^r, \\ \partial_t(AE) + \partial_x\left(Au(E+p)\right) = A(x)uS_u^r, \end{cases} \tag{1}$$

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and the associated unknown is $W = (A\rho, A\rho u, AE)^T$. The terms on the right-hand side S^r_U stand for the *regular* head losses, which account for viscous effects on the walls of the pipe. This set of equations is meant when the cross-section $A(x)$ is regular (at least C^2). In practice, this information is provided by the user. When convergent or divergent regions occur in the pipe, and when the meshes that are used are rather coarse, the well-balanced strategy naturally arises for those who use Finite Volume methods. In that case, the cross section is assumed to be constant within each cell, and another equation is added to (1), which is $\partial_t A = 0$. The idea is very appealing, and many authors have been investigating that particular topic, both at the continuous and discrete levels, among which we may at least cite [2,3,8–10,12]. Though the cross section $A(x)$ is known, the new unknown is now $\tilde{W} = (A, A\rho, A\rho u, AE)^{\mathrm{T}}$, and terms of the form $\psi(W)\partial_x A$ are now part of the convective subset. Meanwhile, a steady wave associated with $\lambda = 0$, corresponding to the "constraint" $\partial_t A = 0$ arises. The one-dimensional Riemann problem associated with system (1) complemented with $\partial_t A = 0$ and discontinuous initial conditions may then be investigated. As a consequence, there is the temptation to use this approach not only to deal with regular cross sections, but also to apply it when discontinuities are present in the pipe, which is obviously of major interest for practical computations in an industrial framework. In that case, the classical well-balanced strategy simply assumes that interface conditions that should be enforced at the steady interface separating two cells with distinct cross sections correspond to the Riemann invariants of the steady wave. In the case of Euler equations, this means that quantities $Q = A\rho u$, QH where $H = (E + p)/\rho$ and the entropy $s(p, \rho)$ should be preserved across the steady interface. Well-Balanced (WB) schemes relying on this approach can then be built, and some of them behave quite well owing to their stability and ability to maintain well-balanced initial conditions, even when the cross section is discontinuous. It even seems that the WB technique is mandatory in order to achieve numerical convergence towards the correct solutions (see for instance [6,11]).

Nonetheless, a major drawback is hidden in the basic formulation. This is due to the fact that for the case of discontinuous cross sections, the physical *singular* head losses have not been accounted for. As a result, the comparison of the "true" multi-dimensional solution and of the one-dimensional well-balanced computational results is rather poor; this is described in [7] for instance. One retrieves the expected results that are the following: the quantities $Q = A\rho u$, QH should remain constant across interfaces supporting discontinuous cross sections, but the entropy does not. This is directly linked to the fact that some singular contribution has been omitted in the momentum equation.

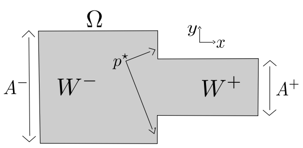
In order to cure that deficiency, at least two strategies may be proposed. A first one simply consists in a modified one-dimensional approach that would rely on an integral formulation of the multi-dimensional set of governing equations. This scheme (called $1D+$ in the sequel) has been partially investigated in [1], and this seems to be a rather promising technique in order to handle this difficulty. Another strategy that is appealing in based on similar ideas, and it simply consists in enforcing a new set of interface conditions at the steady interface associated with the cross-section discontinuity. The basic requirement is that these modified interface conditions should account for singular head losses due to pressure effects, at least in a weak way. This is precisely the main objective of the present contribution.

Thus the paper is organized as follows: we firstly introduce the new interface condition, we secondly explain how to compute left and right states satisfying the new interface condition, we thirdly introduce the numerical scheme and we finally do some numerical validations.

2. A new formulation for flows in variable cross section ducts

2.1. The new interface condition

We omit the regular head losses in System (1), which means that $S^r_u = 0$ in System (1). Integrating the multi-dimensional Euler equations on the physical domain Ω (see Figure below), we may now propose three natural candidates for the *steady* interface conditions



$$\begin{cases} [Q]^+_- = 0, \\ [Qu + Ap]^+_- - p^\star [A]^+_- = 0, \\ [QH]^+_- = 0, \end{cases} \tag{2}$$

$$\text{with } p^\star = \begin{cases} p^-, & \text{if } A^- > A^+, \\ p^+, & \text{otherwise}, \end{cases} \tag{3}$$

where we note by $\cdot^+$ (resp. $\cdot^-$) the value of the quantity on the right (resp. the left) side of the discontinuous interface. The choice of the wall pressure $p^\star$ is based on the two-dimensional approach. Indeed, if we have a contraction with $A^- > A^+$ (resp. an expansion with $A^- < A^+$), the pressure on the restricted wall is the left (resp. right) pressure.

2.2. Computation of left and right states satisfying the interface condition (2)

We assume that the fluid satisfies a perfect gas equation of states: $p = (\gamma - 1)\,\rho\,\epsilon$.

$\underline{W^+ = W^+(W^-)}$: we assume that W^- is known and we compute W^+ such that the interface condition (2) is satisfied. For given value of W^-, we can compute $Q^- = A^-\rho^-u^-$ and $H^- = \frac{\gamma}{\gamma-1}\frac{p^-}{\rho^-} + \frac{(u^-)^2}{2}$. The first two conditions in (2) give