Journal of Family Business Strategy 5 (2014) 97–104

Contents lists available at ScienceDirect

Journal of Family Business Strategy

journal homepage: www.elsevier.com/locate/jfbs

Bayesian methods in family business research

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ARTICLE INFO

Article history:
Received 17 October 2013
Received in revised form 17 December 2013
Accepted 22 December 2013

Keywords:
Bayesian methods
Null hypothesis significance testing
Family business research methods
Regression models

ABSTRACT

Bayesian methods constitute an alternative to null hypothesis significance testing (NHST). This article briefly reviews the concept of Bayesian methods, describes their differences from NHST, and discusses the potential of Bayesian methods to advance family business research and practice. We argue that Bayesian methods are well suited to account for the significant heterogeneity that exists in the population of family firms. The article closes with a short guide to using Bayesian methods and reporting their results in the context of research on family businesses. The article's focus is on regression models.

1. Introduction

Bayesian analysis is a commonly encountered and well-accepted statistical method that is employed in academic disciplines including medicine (Armitag et al., 2009; Ashby, 2006; Berry, 2006), psychology (Edwards, Lindman, & Savage, 1963), physics (Cousins, 1995), genetics (Shoemaker, Painter, & Weir, 1999), and biology (Huelsenbeck, 2001). With the exception of marketing research (Rossi & Allenby, 2003) and decision analysis (Grover, 2013), however, Bayesian methods are rarely utilised in business or management research (Kruschke, Aguinis, & Joo, 2012; Zyphur & Oswald, 2013).[3] A number of recent studies lament this situation and suggest that Bayesian methods may be a useful alternative to null hypothesis significance testing (NHST) (Hahn & Doh, 2006; Hansen, Perry, & Reese, 2004; Kruschke, Aguinis, & Joo, 2012; The Economist, 2006), which has long been the dominant mode of statistical analysis in management research (Schwab, Abrahamson, Starbuck, & Fidler, 2011).[4] This article extends this small but growing literature on the Bayesian approach and examines the potential applicability of Bayesian methods to family business research and practice. The focus of the article is on regression models.

We proceed as follows: Section 2 describes NHST and introduces Bayesian analysis. Section 3 presents the most cogent criticisms of NHST, and Section 4 enumerates the differences between Bayesian analysis and NHST. Section 5 discusses the potential contributions of Bayesian analysis to family business research, and Section 6 provides a short illustrative example of the difference between Bayesian analysis and NHST. Section 7 contains a short guide for how to use Bayesian methods and report results in the context of research on family businesses. Section 8 concludes.

2. NHST and Bayesian analysis

The NHST approach defines a population (e.g., all students in Germany) and draws a sample from this population (e.g., students at the University of Trier) to learn about the value of a particular parameter in the population (e.g., mean age). The assumption is that a parameter varies across the population. One sample taken from the population will yield a particular parameter value, whereas a different sample will yield another value, and the difference between the samples is referred to as sampling variation. The statistician's task is to arrive at the 'true' parameters of the population using the evidence provided by the sample. To accomplish this objective, a sample estimator and an accompanying test statistic are selected. Thus, the NHST approach is tied to the notion of a sample and a population; this approach uses sample estimators and test statistics to learn something about the 'true' parameters in the population. The Bayesian approach is different because it is *not* tied to the notion of a sample and a population.

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[3] The situation in economics is comparable: with the exception of macroeconomics (e.g., Smets & Wouters, 2007), most empirical research in economics is based on NHST.
[4] We found only two articles researching family business using Bayesian methods (Block et al., 2011; Block & Wagner, 2013).

1877-8585/$ – see front matter
http://dx.doi.org/10.1016/j.jfbs.2013.12.003

Bayesian methods rely on Bayes' theorem in probability theory (Bayes, 1763), which is given by:

$$Pr(\theta|y) = \frac{Pr(y|\theta)Pr(\theta)}{Pr(y)} \quad (1)$$

where θ represents the set of unknown parameters and y represents the data. $\mathrm{Pr}(\theta)$ is the prior distribution of the parameter set θ,[5] which may be derived from theory, expert opinion, or other external sources. $Pr(y|\theta)$ is the likelihood function, which is the probability of the data y given the unknown parameter set θ. $\mathrm{Pr}(y)$ is the marginal distribution of the data y; finally, $Pr(y|\theta)$ represents the posterior distribution,[6] which is the probability of the parameter set θ given the data y. Eq. (1) may also be written as:

$$Pr(\theta|y) \propto Pr(y|\theta)Pr(\theta), \quad (2)$$

where $\propto$ indicates 'proportional to'. The posterior distribution is proportional to the likelihood function multiplied by the prior distribution.[7] In Bayesian analysis, inference comes from the posterior distribution, which states the likelihood of a particular parameter value.

When testing a hypothesised relationship between two variables, Bayesian analysis proceeds in the following three steps. First, a priori beliefs (from theory, prior empirical research or an interview) about the relationship of interest are formulated (the prior distribution, $\mathrm{Pr}(\theta)$). Next, a probability of occurrence of the data given these a priori beliefs is assumed (the likelihood function, $Pr(y|\theta)$). In the third step, data are used to update these beliefs. The result is the posterior distribution, $Pr(\theta|y)$). This posterior distribution gives a density function of the parameter of interest (i.e., the coefficient that describes the relationship between the two variables). The posterior distribution allows for statements in terms of likely and unlikely parameter values.

3. Criticism of NHST

Since its introduction by Fisher (1925), NHST has been criticised for myriad reasons (e.g., Cohen, 1994; Schmidt, 1996; Starbuck, 2006).[8] Fisher himself was aware of the problems associated with NHST and recommended its use primarily when researchers have little prior knowledge about the object of their research (Gigerenzer, Krauss, & Vitouch, 2004).

One of the main problems of NHST is argued to be the statistical significance level required for publication (in most cases 5%), which is arbitrary and has no mathematical basis; this statistical significance level is found simply as the result of long tradition (Gigerenzer, Krauss, & Vitouch, 2004). Frequently, researchers applying NHST ignore the fact that the obtained significance level is tied to the test's statistical power and the sample size. In addition, NHST interprets the result of an empirical analysis as dichotomous, i.e., either an empirical result is statistically significant or not. Small differences in data (e.g., a p-value that drops from $p = 0.051$ to $p = 0.048$) can therefore lead to major differences in inference (Schwab et al., 2011) and interpretation of empirical results.[9] A more pedagogical criticism addresses the fact that overstressing statistical significance draws attention from the size of an effect (economic significance) (Combs, 2010). NHST does not distinguish between economically and practically important versus unimportant effects; this judgement is simply left to the researcher. Additionally, NHST is sensitive to the size of the sample, and a statistically significant result can almost always be found if a large enough sample is analysed (Berkson, 1938; Combs, 2010).[10] The outcome of a research project therefore is critically dependent on a researcher' ability to obtain sufficient data (Sawyer & Ball, 1981). Another criticism concerns the interpretation of a result that does not allow the rejection of the null hypothesis (e.g., $p > 0.05$ with a 5% significance level). A non-significant result can result from a small sample size, a violation of assumptions of the specific estimators and statistical tests used, or a non-existing relationship. Finally, scientific journals almost never publish statistically non-significant results and thereby present a biased picture of reality. Meta-analyses, in particular, can suffer from this publication bias (Stanley, 2005).

4. Differences between NHST and Bayesian analysis

The Bayesian approach is fundamentally different from NHST. The main differences are the following:

- *Posterior distribution* instead of a point estimate: As explained in Section 2, the result of Bayesian analysis is a posterior distribution of the parameter of interest, which differs from NHST in that the outcome of the estimation is not a point estimate (i.e., whether a value is either statistically significant or not) but an entire distribution function. Thus, Bayesian analysis allows for statements such as "the probability of a positive effect of A on B is 70%", which is not possible with NHST. NHST only permits a statement such as "the effect of A on B is positive. The probability of making an error with this statement is below 5% (10%)."
- *Notion of sample and population*: Bayesian analysis is *not* tied to the notion of a sample and a population. Its results are statements about the particular data that are used in the analysis. Thus, there is no statement about a 'true' parameter in an underlying distribution. When Bayesians refer to their data as a sample, it is simply out of convention.
- *Prior*: A Bayesian researcher must formulate an assumption about the distribution function of the coefficient of interest. This so-called prior (probability) can be either subjective and informative or objective and minimally informative. Assume that a researcher believes that variable A has a positive influence on variable B. A subjective (informative) prior would refer to a probability distribution function with a positive mean and few or no values smaller than zero; an objective (minimally informative) prior would refer to a flat distribution function or a Gaussian distribution with a mean of zero.[11] In most cases, a Bayesian researcher should investigate the sensitivity of the results that are obtained to the specification of the prior.
- *Likelihood function*: In addition to specifying the prior, a Bayesian[12] researcher must attach probabilities to the values of the data observed. As explained in Section 2, attaching the

[5] This distribution is often referred to as the *prior*.

[6] This distribution is often referred to as the *posterior*.

[7] The denominator $\mathrm{Pr}(y)$ can be neglected in Bayesian estimation; $\mathrm{Pr}(y)$ is constant and often unknown and it is also independent from the parameters of the model.

[8] The criticism has been most intense in the field of psychology. The American Psychological Association (APA) reacted and enhanced their publication guidelines by reducing the relative importance of NHST. Reporting confidence intervals for effect sizes has become standard.

[9] It is notable that this overstressing of a drop in the p-value is more of a problem regarding the application of NHST (and the interpretation of the p-value) than of the statistic itself (Hubbard & Bajarri, 2003).

[10] By definition, one in 10 studies will produce a significant result ($p < 10\%$, two-sided test). If this study is then published and the other studies are not published, the uninformed reader will conclude that there is a significant relationship in the population.

[11] The researcher can also formulate a prior opposite to her expectations. This type of prior formulation is frequently used as a robustness check. An empirical result is considered particularly robust if the evidence (the data) is able to "correct" a prior formulated against the researcher's beliefs.

[12] People who follow Bayesian methods are sometimes called Bayesians. People who follow NHST are sometimes referred to as 'frequentists'.