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Degree of locally condensing perturbations of Fredholm maps with positive index and applications ☆

Victor Zvyagin [a], Nadezhda Ratiner [b,*]

[a] Mathematical Faculty, Voronezh State University, Voronezh, Russia
[b] Physical Faculty, Voronezh State University, Voronezh, Russia

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ABSTRACT

We give a construction of the topological degree for locally condensing perturbations of C^1-Fredholm maps of positive index with values in the GL_c-framed cobordism group. The construction of the degree extends constructions of the degree for Fredholm maps of positive index and also its finite-dimensional, compact, and compactly restricted perturbations. It based on the finite-dimensional reduction method. We apply the constructed degree to prove the existence theorem for a system of ordinary differential equations with Hopf's boundary conditions.

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* Corresponding author: Universitetskaya pl. 1, Voronezh State University, Physical Faculty, Department of Mathematical Physics, 394693, Russia.

E-mail addresses: zvg@math.vsu.ru (V. Zvyagin), ratiner@phys.vsu.ru (N. Ratiner).

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Contents

1. Introduction

In 1965 S. Smale published his famous paper [21]. In this paper he presented an infinite dimensional version of Sard's theorem and defined a nonoriented degree for nonlinear Fredholm maps with values in nonoriented cobordism group. The degree of S. Smale is defined both for Fredholm maps of zero and positive index. However, the nonoriented cobordism group for some dimensions is not rich enough, in fact for Fredholm maps of index zero the degree of Smale is simply the modulo two degree, for Fredholm maps of indexes 1 and 3 this degree is always zero. This initiated the development of the oriented degree theory for Fredholm nonlinear maps. In [7], [8] K.D. Elworthy and A.J. Tromba defined Fredholm structures on Banach manifolds, which is the natural framework for the generalization of the notion of orientation for the infinite-dimensional case. Using the introduced notion of orientation and the Sard-Smale theorem they defined the integer valued oriented degree for Fredholm maps of index zero. To define the degree for Fredholm maps of positive index they extended the Pontrjagin construction of framed cobordism group [16] to infinite-dimensional case, defined the GL_c-famed cobordism groups of Fredholm structure, and the degree for Fredholm maps of positive index with values in these groups.

The construction in [7], [8], both for maps of index zero and of positive index, requires the high smoothness of involved maps and manifolds, and also requires existence of the smooth partition of unity. This narrows the area of possible applications of the above degree.

It was the reason for development of the degree theory for a wider class of Fredholm maps. Many different constructions of degree appeared, especially for zero index Fredholm maps, see, e.g., [4], [10], [11], [9], [17], [15], [2], [3].