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NONLINEAR PROFILE DECOMPOSITION FOR THE $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ ENERGY SUBCRITICAL WAVE EQUATION

Javier Ramos

Abstract

We consider the nonlinear wave equation $u_{tt} - \Delta u \pm u|u|^{\frac{4}{d-1}} = 0$ in dimensions $d \geq 2$. This equation is critical in $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ and subcritical with respect to the energy. We prove the nonlinear profile decomposition. The proof must address the lack of compactness due to the Lorentz symmetry.

1 Introduction

We consider the initial value problem for the nonlinear wave equation,

$$\begin{cases} u_{tt} - \Delta u + \gamma u|u|^{\frac{4}{d-1}} = 0 \\ u(0) = u_0 \in \dot{H}^s, \quad u_t(0) = u_1 \in \dot{H}^{s-1}, \end{cases} \quad (1)$$

where $\gamma \in \{\pm 1\}$, $s = \frac{1}{2}$ and $d \geq 2$. This equation is critical in $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ due to the scaling, which leaves the equation invariant

$$u_r(x, t) = r^{\frac{d-1}{2}} u(rx, rt),$$

and also preserves the $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ norm

$$\|(u_r(0), \partial_t u_r(0))\|_{\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)} = \|(u(0), \partial_t u(0))\|_{\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)}.$$

The problem is energy subcritical because the regularity associated to the conserved energy,

$$E(u) = \int_{\mathbb{R}^d} \frac{1}{2} |\partial_t u|^2 + \frac{1}{2} |\nabla u|^2 + \gamma \frac{1}{2} \frac{d-1}{(d+1)} |u|^{\frac{d+1}{d-1}} dx,$$

lies in $\dot{H}^1 \times L^2(\mathbb{R}^d)$.

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