Nonlinear profile decomposition for the $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ energy subcritical wave equation

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PII: S0022-1236(18)30326-4
DOI: https://doi.org/10.1016/j.jfa.2018.08.023
Reference: YJFAN 8093

To appear in: *Journal of Functional Analysis*

Received date: 14 April 2017
Accepted date: 31 August 2018

NONLINEAR PROFILE DECOMPOSITION FOR THE $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ ENERGY SUBCRITICAL WAVE EQUATION

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Abstract

We consider the nonlinear wave equation $u_{tt} - \Delta u \pm u|u|^{\frac{4}{d-1}} = 0$ in dimensions $d \geq 2$. This equation is critical in $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ and subcritical with respect to the energy. We prove the nonlinear profile decomposition. The proof must address the lack of compactness due to the Lorentz symmetry.

1 Introduction

We consider the initial value problem for the nonlinear wave equation,

$$\begin{cases} u_{tt} - \Delta u + \gamma u|u|^{\frac{4}{d-1}} = 0 \\ u(0) = u_0 \in \dot{H}^s, \ u_t(0) = u_1 \in \dot{H}^{s-1}, \end{cases} \tag{1}$$

where $\gamma \in \{\pm 1\}$, $s = \frac{1}{2}$ and $d \geq 2$. This equation is critical in $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ due to the scaling, which leaves the equation invariant

$$u_r(x,t) = r^{\frac{d-1}{2}} u(rx, rt),$$

and also preserves the $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)$ norm

$$\|(u_r(0), \partial_t u_r(0))\|_{\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)} = \|(u(0), \partial_t u(0))\|_{\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}(\mathbb{R}^d)}.$$

The problem is energy subcritical because the regularity associated to the conserved energy,

$$E(u) = \int_{\mathbb{R}^d} \frac{1}{2}|\partial_t u|^2 + \frac{1}{2}|\nabla u|^2 + \gamma \frac{1}{2}\frac{d-1}{(d+1)}|u|^{2\frac{d+1}{d-1}} dx,$$

lies in $\dot{H}^1 \times L^2(\mathbb{R}^d)$.

ICMAT, Madrid. Post-doctoral grant financed by Severo Ochoa Excellence Programme. Partially supported by MICINN grants MTM2013-40945-P and MTM2017-85934-C3-1-P, ERC grant 277778 and CAPES-Brazil.
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