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# Fourier coefficients attached to small automorphic representations of $\mathrm{SL}_n(\mathbb{A})$

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We show that Fourier coefficients of automorphic forms attached to minimal or next-to-minimal automorphic representations of $\mathrm{SL}_n(\mathbb{A})$ are completely determined by certain highly degenerate Whittaker coefficients. We give an explicit formula for the Fourier expansion, analogously to the Piatetski-Shapiro–Shalika formula. In addition, we derive expressions for Fourier coefficients associated to all maximal parabolic subgroups. These results have potential applications for scattering amplitudes in string theory.



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Nilpotent orbits
String Theory
Scattering amplitudes

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## 1. Introduction

### 1.1. Background and motivation

Let $F$ be a number field and $\mathbb{A}$ be the associated ring of adeles. Let G be a reductive algebraic group defined over $F$ and $\pi$ an (irreducible) automorphic representation of $\mathrm{G}(\mathbb{A})$ as defined in [BJ79,FGKP18].

Fix a Borel subgroup $B$ and let $P \subset \mathrm{G}$ be a standard parabolic subgroup with Levi decomposition $P = LU$, and let $\psi : U(F)\backslash U(\mathbb{A}) \to \mathbb{C}^\times$ be a global unitary character. Given any automorphic form $\varphi \in \pi$ one can consider the following function on $\mathrm{G}(\mathbb{A})$:

$$\mathcal{F}_U(\varphi, \psi; g) = \int\limits_{U(F)\backslash U(\mathbb{A})} \varphi(ug)\psi^{-1}(u)\, du\,.$$

This can be viewed as a Fourier coefficient of the automorphic form $\varphi$ with respect to the unipotent subgroup $U$. Fourier coefficients of automorphic forms carry a wealth of arithmetic and representation-theoretic information. For example, in the case of classical modular forms on the upper half-plane, Fourier coefficients are well-known to encode information about the count of rational points on elliptic curves. On the other hand, for higher rank Lie groups their arithmetic content is not always transparent, but they always encode important representation-theoretic information. Langlands showed that the constant terms in the Fourier expansion of Eisenstein series provide a source for