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Indivisibility of divisor class numbers of Kummer extensions over the rational function field

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ABSTRACT

We find a complete criterion for a Kummer extension K over the rational function field $k = \mathbb{F}_q(T)$ of degree ℓ to have indivisibility of its divisor class number h_K by ℓ, where $\mathbb{F}_q$ is the finite field of order q and ℓ is a prime divisor of $q-1$. More importantly, when h_K is not divisible by ℓ, we have $h_K \equiv 1 \pmod{\ell}$. In fact, the indivisibility of h_K by ℓ depends on the number of finite primes ramified in K/k and whether or not the infinite prime of k is unramified in K. Using this criterion, we explicitly construct an infinite family of the maximal real cyclotomic function fields whose divisor class numbers are divisible by ℓ.

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1. Introduction

Let $k = \mathbb{F}_q(T)$ be the rational function field, where $\mathbb{F}_q$ is the finite field of order q and q is a power of a prime p. There have been active developments on the divisor class

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numbers of global function fields. For instance, Ichimura [4] constructed infinitely many imaginary quadratic extensions over k whose divisor class numbers are not divisible by 3. Byeon [1] extended his result to the case of their indivisibility by odd prime ℓ with $\ell \neq p$. Furthermore, Pacelli and Rosen [6] extended Ichimura's result to algebraic function fields which are not necessarily quadratic over k in terms of their indivisibility by 3. Furthermore, in the case that $q \equiv -1 \pmod{\ell}$, Daub et al. [2] showed, by nonconstructive proof, the existence of infinitely many function fields of degree m over k whose divisor class numbers are not divisible by an odd prime ℓ, where m is a positive integer divisible by ℓ. In this work, we focus on the case that $q \equiv 1 \pmod{\ell}$ and ℓ is any prime.

We find a complete criterion for a Kummer extension K over the rational function field $k = \mathbb{F}_q(T)$ of degree ℓ to have indivisibility of its divisor class number h_K by ℓ, where $\mathbb{F}_q$ is the finite field of order q and ℓ is a prime divisor of $q-1$. More importantly, when h_K is not divisible by ℓ, we have $h_K \equiv 1 \pmod{\ell}$. In fact, the indivisibility of h_K by ℓ depends on the number of finite primes ramified in K/k and whether or not the infinite prime of k is unramified in K; Theorem 1.1 (respectively, Theorem 1.2) is for the case that the infinite prime of k is ramified (respectively, unramified) in K. Using this criterion, we explicitly construct an infinite family of the maximal real cyclotomic function fields whose divisor class numbers are divisible by ℓ (Theorem 1.3).

We use the following notation throughout the paper.

Notation

q	a prime power
ℓ	a prime divisor of $q-1$
$P_i = P_i(T)$	an irreducible monic polynomial in $\mathbb{F}_q[T]$ for every i
$Q(T)$	$aP_1^{e_1}P_2^{e_2}\cdots P_t^{e_t}$, where $a \in \mathbb{F}_q^*$ and $1 \leq e_i \leq \ell - 1$
$k = \mathbb{F}_q(T)$	the rational function field
$K = k(\sqrt[\ell]{Q(T)})$	a Kummer extension of degree ℓ
t	the number of finite primes of k which are ramified in K
∞	the infinite prime of k
g	the genus of K
d_i	the degree of $P_i(T)$ for i with $1 \leq i \leq t$
δ	the degree of $Q(T)$
δ_0	$\sum_{i=1}^{t} \deg P_i(T)$
h_K	the divisor class number of K
a_n	the number of prime divisors of K with degree n
b_n	the number of effective divisors of K with degree n
$k(\Lambda_P)$	the Pth cyclotomic function field
$k(\Lambda_P)^+$	the maximal real subfield of $k(\Lambda_P)$

We state the main results as follows.

Theorem 1.1. *Let K be a Kummer extension over the rational function field $k = \mathbb{F}_q(T)$ of degree ℓ, where $\mathbb{F}_q$ is the finite field of order q and ℓ is a prime divisor of $q-1$. Assume that the infinite prime ∞ of k is ramified in K.*

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