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MENON-TYPE IDENTITIES WITH ADDITIVE CHARACTERS

YAN LI AND DAEYEOUL KIM*

ABSTRACT. The classical Menon's identity [7] states that

$$\sum_{\substack{a=1 \\ \gcd(a,n)=1}}^n \gcd(a-1, n) = \varphi(n)\tau(n),$$

for every positive integer n , where $\varphi(n)$ is the Euler's totient function and $\tau(n)$ is the number of positive divisors of n . Recently, Zhao and Cao [19] extended Menon's identity to Dirichlet characters :

$$\sum_{\substack{a=1 \\ \gcd(a,n)=1}}^n \gcd(a-1, n)\chi(a) = \varphi(n)\tau\left(\frac{n}{d}\right),$$

where χ is a Dirichlet character modulo n and d is the conductor of χ . In this paper, we extend Menon's identity to additive characters. A special case of our main result reads like this :

$$\sum_{\substack{a=1 \\ \gcd(a,n)=1}}^n \gcd(a-1, n) \exp\left(\frac{ka2\pi i}{n}\right) = \exp\left(\frac{k2\pi i}{n}\right) \tau(\gcd(k, n))\varphi(n)$$

if $\text{ord}_p(n) - \text{ord}_p(k) \neq 1$ holds for any prime p dividing n , where for $u \in \mathbb{Z}$, $\text{ord}_p(u)$ is the exponent of the highest power of p dividing u . For $k = 0$, our result reduces to the classical Menon's identity.

1. INTRODUCTION

Throughout this paper \mathbb{N} and \mathbb{Z} will denote sets of positive integers and integers, respectively. In 1965, P. K. Menon [7] proved the following elegant identity

$$(1) \quad \sum_{\substack{a=1 \\ \gcd(a,n)=1}}^n \gcd(a-1, n) = \varphi(n)\tau(n),$$

valid for every positive integer n , where $\gcd(\cdot, \cdot)$ represents the greatest common divisor, φ is the Euler's totient function and $\tau(n)$ is the number of positive divisors of n .

Many authors gave various generalizations of Menon's identity in different directions. For example, Sury [12] obtained the following Menon-type identity

$$(2) \quad \sum_{\substack{1 \leq a_1, \dots, a_s \leq n \\ \gcd(a_1, n)=1}} \gcd(a_1 - 1, a_2, \dots, a_s, n) = \varphi(n)\sigma_{s-1}(n),$$

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