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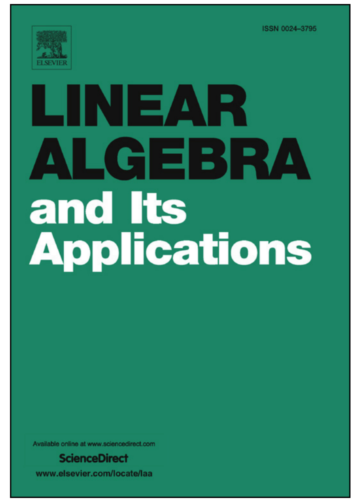

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JOINT SIMILARITY FOR COMMUTING FAMILIES OF POWER BOUNDED MATRICES

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ABSTRACT. An example due to Pisier shows that two commuting, completely polynomially bounded Hilbert space operators may not be simultaneously similar to contractions. Thus, while each operator is individually similar to a contraction, the pair is not *jointly* similar to a pair of commuting contractions. We show that this phenomenon does not occur in finite dimensions. More precisely, we show that a finite family of power bounded commuting matrices is always jointly similar to a family of contractions. In fact, the result can be extended to infinite families satisfying certain uniformity conditions. Our approach is based on a joint spectral decomposition of the underlying space.

1. INTRODUCTION

The classical von Neumann inequality [17] states that for a contractive linear operator T acting on a Hilbert space, we always have that

$$\|f(T)\| \leq \sup_{z\in\mathbb{D}} |f(z)|$$

for every polynomial f, where $\mathbb{D}$ denotes the open unit disc in the complex plane. This observation lies at the base of the fruitful connection between complex function theory and operator theory. It also provides motivation for one of Halmos' famous ten problems [6], essentially asking to characterize the class of *polynomially bounded* operators, that is those Hilbert space operators for which von Neumann's inequality holds up to a multiplicative constant. It is readily seen that being similar to a contraction is a sufficient condition for an operator to be polynomially bounded, and Halmos asked whether this condition was in fact necessary. If the condition is weakened to the operator merely having uniformly bounded powers, then this was shown not to be the case by Foguel [5].

A key insight into Halmos' question was provided by Paulsen [9], who showed that an operator is similar to a contraction if and only if it is *completely* polynomially bounded, in the sense that it satisfies von Neumann's inequality up to a multiplicative constant for arbitrary matrix-valued polynomials. Such a characterization turned out to be very fruitful, and led to the solution of Halmos' problem by Pisier [11]. Therein, an example is exhibited of a polynomially bounded operator which is not completely polynomially bounded. A somewhat streamlined treatment appears in [3]. We refer the interested reader to [8, Chapter 10] or [13, Chapter 28] for a detailed account of this problem and its solution.

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