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Global dynamics of a mechanical system with dry friction

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Abstract

The global dynamics of a mechanical system with dry friction is completely analyzed. This mechanical system is a class of discontinuous and transcendental piecewise smooth differential systems. Moreover, it can exhibit rich and complex dynamical phenomena, such as Hopf bifurcation, grazing bifurcation, grazing-sliding bifurcation and bifurcations of limit cycles. Finally, all global phase portraits of the system are presented on the Poincaré disc.

Keywords: Global phase portrait; Limit cycle; Grazing bifurcation; Grazing-sliding bifurcation; Discontinuous piecewise differential system

1. Introduction

Friction is a common phenomenon in nature. When two surfaces come into contact, forces are produced by each surface on the other. The part that is tangent to the contacting surfaces is called the frictional force, which resists the relative motion of solid surfaces, fluid layers, and

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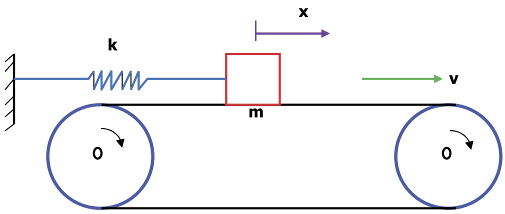
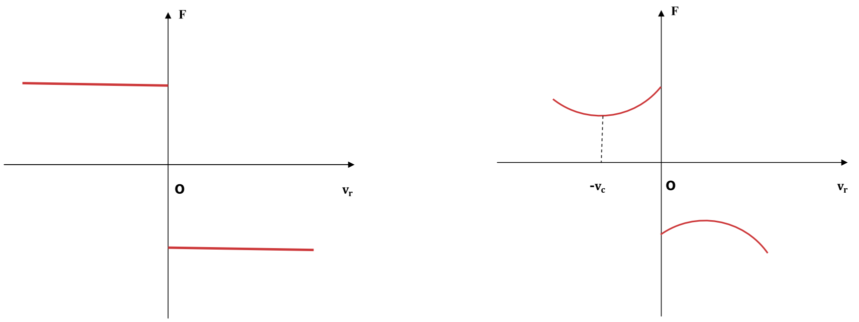


Fig. 1. A belt pulley with dry friction.



(a) Piecewise linear shape (b) Piecewise smooth shape

Fig. 2. Different shapes of the friction characteristic.

material elements sliding against each other. There are several types of friction, among which dry friction describes the reaction between two solid bodies in contact with each other when they are in motion (kinetic friction) and when they are not (static friction). More introductions of friction are provided in [24] and [27].

A single-degree-of-freedom model with dry friction was considered in [9–11,23]

$$\dot{x} = y, \qquad \dot{y} = -\frac{k}{m}x - cy + \frac{F(y-v)}{m}, \tag{1}$$

which exhibits the dynamical behavior of an oscillator consisting of a mass m, supported by a moving belt, and connected to a fixed support by a linear elastic spring and by a linear dashpot, as shown in Fig. 1. Here, the block mass m, the damping coefficient c, the stiffness of the spring k, and the speed v of the belt are positive constants.

The kinetic friction force has different forms. According to [9], in dimensionless terms, the simplest kinetic friction is given by the following function

$$F(v_r) = \frac{\mu\,\mathrm{sgn}(v_r)}{m}, \tag{2}$$

where $0 < \mu < 1$ and $v_r = y - v$; see Fig. 2(a). This model was first considered in [23], where the kinetic friction is a constant force opposite to the relative motion between the block and the belt, with a smaller magnitude than the maximum static friction force F_s. In dimensionless terms, $F_s = 1$.

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