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Existence of global weak solutions for the Navier–Stokes–Vlasov–Boltzmann equations

Lei Yao [a], Cheng Yu [b,*]

[a] *School of Mathematics and Center for Nonlinear Studies, Northwest University, Xi'an 710127, PR China*
[b] *Department of Mathematics, The University of Texas at Austin, TX 78712, USA*

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Abstract

The motion of moderately thick spray can be modeled by a coupled system of equations consisting of the incompressible Navier–Stokes equations and the Vlasov–Boltzmann equation. In this paper, we study the initial value problem for the Navier–Stokes–Vlasov–Boltzmann equations. The existence of global weak solutions is established by the weak convergence method.

MSC: 35Q30; 35Q99

Keywords: The moderately thick spray; Breakup kernel; Navier–Stokes–Vlasov–Boltzmann equations; Existence; Weak solution

1. Introduction

A spray is a process in which drops are dispersed in a gas. Spray can help people to distribute material over a cross-section and to generate liquid surface area. There are various applications in different fields: including but not limited to, fuel injectors for gasoline and Diesel engines, atomizers for jet engines, atomizers for injecting heavy fuel oil into combustion air in steam boiler injectors, and rocket engine injectors. It also has a big impact on crop yields, plant health, efficiency of pest control and of course, profitability.

* Corresponding author.
E-mail addresses: yaolei1056@hotmail.com (L. Yao), yucheng@math.utexas.edu (C. Yu).

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For a moderately thick spray, we can assume that the volume fraction occupied by the droplets is small enough to be neglected. Thus, we are able to apply the Vlasov–Boltzmann equation to model the liquid phase, which can be performed by a particle density function. In particular, function $f(t,x,\xi)$ denotes a number density of droplets of which at time t and physical position x with velocity ξ. It is a solution of the following Vlasov–Boltzmann equation

$$f_t+\xi\cdot\nabla_x f+\nabla(f\mathfrak{F})=Q(f,f), \tag{1.1}$$

where $\mathfrak{F}$ is an acceleration resulting from the drag force exerted by the gas, and $Q(f,f)$ is an operator taking into account complex phenomena happening at the level of the droplets (collisions, coalescences, breakup).

The incompressible Navier–Stokes equations could be used to describe the motion of gas:

$$\begin{aligned} &u_t+u\cdot\nabla u+\nabla P-\mu\Delta u=F_e,\\ &\mathrm{div}u=0, \end{aligned} \tag{1.2}$$

where u is the velocity of the gas, and the external force

$$F_e=-c\int_{\mathbb{R}^3} f\mathfrak{F}\,d\xi, \tag{1.3}$$

where acceleration $\mathfrak{F}$ is given by

$$\mathfrak{F}=-\frac{9\mu}{2\rho}\frac{\xi-u}{r^2},$$

μ is the viscosity constant of the incompressible Navier–Stokes equations, ρ is the density of gas, r is the radius of the droplet. We can assume that $\frac{9\mu}{2\rho r^2}=1$ in the whole paper, thus

$$\mathfrak{F}=-(\xi-u). \tag{1.4}$$

One of the typical forms of the collision kernel is given by

$$Q(f,f)=-\lambda f(t,x,\xi)+\lambda\int_{\mathbb{R}^3} T(\xi,\xi')f(t,x,\xi')\,d\xi', \tag{1.5}$$

where $\lambda>0$ is the breakup frequency. The kernel $T(\xi,\xi')$ is the probability of a change with respect to velocity from ξ' to ξ, and ξ is the velocity of individuals before the collision while ξ' is the velocity immediately after the collision. Given that a reorientation occurs, the probability function $T(\xi,\xi')$ is a non-negative function and after normalization we have

$$\int_{\mathbb{R}^3} T(\xi,\xi')d\xi=1. \tag{1.6}$$

As a result, we arrive at a model for a moderately thick spray, namely the Navier–Stokes–Vlasov–Boltzmann equations:

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