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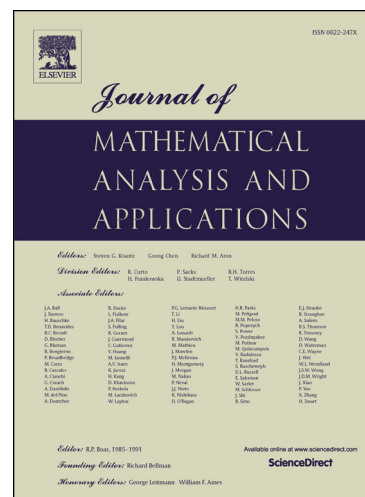
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Proof of the Deficiency Index Conjecture

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ABSTRACT. We prove that all possible values of the deficiency indices r, s of a symmetric (formally self-adjoint) differential expression M with complex coefficients which satisfy the well known inequalities are realized. Subject to these inequalities, in 1974 Kogan and Rofe-Beketov showed that all values of r, s which differ by no more than 1 can be realized, in 1978, 1979 Gilbert showed that their difference can be arbitrarily large provided the order of M is large enough. In this paper we show that all values of r and s are realized.

1. Introduction

For a symmetric expression M and a weight function w , the deficiency indices d^+, d^- , are defined as the number of linear independent solutions of the equation

$$(1.1) \quad My = \lambda wy \text{ on } J, \lambda \in \mathbb{C}$$

which are in $H = L^2(J, w)$. Let \mathbb{C} denote the complex numbers and \mathbb{R} the real numbers. The deficiency indices depend on M, w, J and λ .

THEOREM 1. *For fixed (M, w, J) , $d(\lambda_1) = d(\lambda_2)$ if $\text{Im}(\lambda_j) > 0$ or if $\text{Im}(\lambda_j) < 0$ for $j = 1, 2$.*

PROOF. This follows from a well known abstract result. The proof given on p. 33 of Naimark's book [12, Theorem 5, p.33] can readily be adapted to the much more general class of symmetric expressions M constructed in Section 2 below. \square

DEFINITION 1. *Let $d^+ = d^+(M, w, J) = d^+(i)$ and $d^- = d^-(M, w, J) = d^-(-i)$.*

What are the possible values of d^+, d^- ? This is the question we answer in this paper.

It is clear from the definition that

$$(1.2) \quad 0 \leq d^+, d^- \leq n$$

where n is the order of M since equation (1.1) cannot have more than n linearly independent solutions.

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