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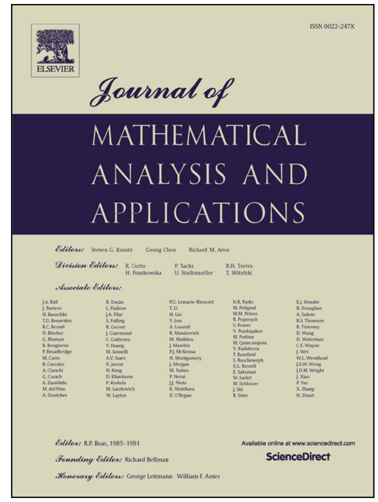

Variational Principles of Hitting Times for Non-reversible Markov Chains

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Variational Principles of Hitting Times for Non-reversible Markov Chains

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Abstract

We give some new kinds of variational formulas for the first hitting time of non-reversible Markov chain on countable state space. Some comparison theorems are obtained for the non-reversible Markov chain and its corresponding reversible one. As an application, we prove a stronger version of a conjecture in [1, Chapter 9, Conjecture 22].

Keywords and phrases: Variational formula, Markov chain, non-reversible, first hitting time, fundamental matrix.

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1 Introduction

Hitting times play an important role in the theory for Markov processes. Especially for Markov chains, the concept of hitting time is the start point to study recurrence, various ergodicity etc.. Refer to [14] for discrete time Markov chains and to [2] for continuous time Markov chains. In recent years, hitting times are used to derive the convergence rate for a Markov process toward its limiting distribution, cf. [3, 18]. In this paper, we give some new kinds of variational formulas for the first hitting time of a non-reversible Markov chain on a countable state space. The existing variational formulas were mainly for the symmetric Markov processes, see [12] for symmetric diffusion processes, [1] for finite symmetric Markov chains. Very recently, in [8] we give a variational formula for finite asymmetric Markov chains, via that of the capacity for asymmetric Markov chains in [6]. As pointed out in [10, Section 3], hitting times are adopted to be an important criterion to show advantages of non-reversible Markov chains, and there are no general results in this direction.

Let V be a countable state space and $Q = (q_{ij} : i, j \in V)$ be an irreducible, totally stable and conservative Q-matrix. That is, for $i \neq j$, there exist distinct $i_1, ..., i_m \in V$

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