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Note on global regularity of 2D regularized MHD equations with zero magnetic diffusion

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we establish the global regularity of the Cauchy problem of the two-dimensional regularized incompressible magnetohydrodynamics equations with the Laplacian dissipation in the velocity equation and zero magnetic diffusion.

1. Introduction and main results

The standard two-dimensional (2D) incompressible magnetohydrodynamics (MHD) equations with only velocity dissipation read as follows

$$\begin{cases} \partial_t v + (v \cdot \nabla) v - \Delta v + \nabla p = (b \cdot \nabla) b, \quad x \in \mathbb{R}^2,\ t > 0, \\ \partial_t b + (v \cdot \nabla) b = (b \cdot \nabla) v, \\ \nabla \cdot v = \nabla \cdot b = 0, \end{cases} \tag{1.1}$$

where $v = (v_1, v_2)$ denotes the velocity, and $b = (b_1, b_2)$, p is the magnetic field and scalar pressure of the fluid respectively. The above MHD system (1.1) can be applied to model plasmas when the plasmas are strongly collisional, or the resistivity due to these collisions are extremely small (see, e.g., [3,4]). Recently, the global well-posedness issue of the system (1.1) has attracted much interest and considerable results have been obtained (see, e.g., [14,15,8,19,23]). However, whether or not the smooth solutions of the above system (1.1) develop finite time singularities is still an outstanding open problem. The main difficulty arising here is a strong impact of the higher modes to the leading order dynamics through the nonlinearity which a priori may destroy the regularity of a solution and thus lead to the formation of singularities. To overcome this difficulty, lots of models have been considered in order to capture the leading dynamics of the flow on

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the one hand and somehow suppress the higher modes on the other hand, see [2,6,5,9,7,18,16,11,12,22,24] for the so-called Leray-alpha models. In this direction, the goal of this paper is devoted to studying the Cauchy problem of the following 2D regularized MHD equations with only velocity dissipation

$$\begin{cases} \partial_t v + (u \cdot \nabla) v - \Delta v + \nabla p = (b \cdot \nabla) b, \quad x \in \mathbb{R}^2,\ t > 0, \\ \partial_t b + (v \cdot \nabla) b = (b \cdot \nabla) u, \\ v = u - \alpha^2 \Delta u, \\ \nabla \cdot u = \nabla \cdot v = \nabla \cdot b = 0, \end{cases} \tag{1.2}$$

subjected to the initial data

$$v(x,0) = v_0(x), \qquad b(x,0) = b_0(x),$$

where $v = (v_1,\, v_2)$ denotes the velocity vector, $b = (b_1,\, b_2)$ the magnetic field, $u = (u_1,\, u_2)$ the "filtered" velocity and p the scalar "filtered" pressure, respectively. $\alpha > 0$ is the length scale parameters that represent the width of the filters. $v_0(x)$ and $b_0(x)$ are the given initial data satisfying $\nabla \cdot v_0 = \nabla \cdot b_0 = 0$. When $\alpha = 0$, the system (1.2) reduces to (1.1). To the best knowledge of the author, the above system (1.2) has not been investigated. To gain further understanding of the global regularity problem for the system (1.1), the goal of this paper is to establish the global regularity for the system (1.2), which can be stated as follows.

Theorem 1.1. *Let $\alpha > 0$ and $(v_0,\, b_0) \in H^s(\mathbb{R}^2)$ $(s > 2)$ with $\nabla \cdot v_0 = \nabla \cdot b_0 = 0$, then the system (1.2) admits a unique global regular solution $(v,\, b)$ such that for any given $T > 0$,*

$$v \in L^\infty([0,T]; H^s(\mathbb{R}^2)) \cap L^2([0,T]; H^{s+1}(\mathbb{R}^2)), \quad b \in L^\infty([0,T]; H^s(\mathbb{R}^2)).$$

The method of proving Theorem 1.1 may also be adapted with almost no change to derive the global regularity of the following 2D regularized MHD system

$$\begin{cases} \partial_t v + (v \cdot \nabla) u - \Delta v + \nabla p = (b \cdot \nabla) b, \quad x \in \mathbb{R}^2,\ t > 0, \\ \partial_t b + (v \cdot \nabla) b = (b \cdot \nabla) u, \\ v = u - \alpha^2 \Delta u, \\ \nabla \cdot u = \nabla \cdot v = \nabla \cdot b = 0. \end{cases} \tag{1.3}$$

More precisely, we have

Theorem 1.2. *Let $\alpha > 0$ and $(v_0,\, b_0) \in H^s(\mathbb{R}^2)$ $(s > 2)$ with $\nabla \cdot v_0 = \nabla \cdot b_0 = 0$, then the system (1.3) admits a unique global regular solution $(v,\, b)$ such that for any given $T > 0$,*

$$v \in L^\infty([0,T]; H^s(\mathbb{R}^2)) \cap L^2([0,T]; H^{s+1}(\mathbb{R}^2)), \quad b \in L^\infty([0,T]; H^s(\mathbb{R}^2)).$$

Remark 1.1. The proof of Theorem 1.2 is largely the same as Theorem 1.1 with only small modifications, thus the details are omitted.

Remark 1.2. At present, we are unable to establish the global regularity result for the system (1.2) and the system (1.3) in higher dimensional case, for example the 3D case.