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Multistability analysis of delayed quaternion-valued neural networks with nonmonotonic piecewise nonlinear activation functions

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ABSTRACT

This paper deals with the multistability problem of the quaternion-valued neural networks (QVNNs) with nonmonotonic piecewise nonlinear activation functions and unbounded time-varying delays. By virtue of the non-commutativity of quaternion multiplication resulting from Hamilton rules, the QVNNs can be separated into four real-valued systems. By using the fixed point theorem and other analytical tools, some novel algebraic criteria are established to guarantee that the QVNNs can have 5^{4n} equilibrium points, 3^{4n} of which are locally μ-stable. Some criteria that guarantee the multiple exponential stability, multiple power stability, multiple log-stability, multiple log–log-stability are also derived as special cases. The obtained results reveal that the introduced QVNNs in this paper can have larger storage capacity than the complex-valued ones. Finally, one numerical example is presented to clarify the validity of the theoretical results.

1. Introduction

In 1843, based on the long-term study on the complex numbers, the Irish mathematician W. R. Hamilton proposed the concept of quaternion. However, the quaternion didn't cause much attention for quite a long time, because quaternion multiplication does not satisfy the commutative law. Quaternion has got a revival since the 1990s, primarily due to its utility in describing spatial rotations [1–5]. The quaternion, which is a significant achievement in algebra, has recently received increasing attention for its effective applications in adaptive filtering [6], computer graphics [7], robotics [8], array processing [9], color image processing [10], etc. There is no doubt that the quaternion gradually shows its advantages in practical applications.

Over the past few decades, the investigation on dynamic behaviors of neural networks has gained in popularity and plentiful results have been obtained for their useful applications in associative memory, pattern recognition, image processing, and so on (see [11–14] and references therein). As is well-known, the two-dimensional data can be well processed by complex-valued neural networks (CVNNs) or many real-valued neural networks (RVNNs). Whereas if the data is three or four dimensional, such as color images, body images, 4-D signals, the quaternion-valued neural networks (QVNNs), which directly encode in terms of quaternions [15], can be more efficient than CVNNs or RVNNs. Moreover, in practical applications

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such as spatial rotation, color night vision, color image compression, and so on [16–19], QVNNs can have unique advance in efficient information processing. Therefore, it is necessary and important to investigate the QVNNs.

As an extension of CVNNs, QVNNs with quaternion-valued states, connections weights and activation functions have gradually aroused researchers' interests, and many significative results have been obtained in recent years [20–25]. The authors studied the discrete-time QVNNs with linear threshold activation functions in [20]. By virtue of characteristic equation, Lyapunov functional and M-matrix, some sufficient conditions to guarantee the boundedness and global exponential periodicity of the QVNNs were derived. In [21,22], by separating the proposed QVNNs into four real-valued systems, the global exponential stability of the delayed QVNNs was investigated. In [23,24], by separating QVNNs into two complex-valued neural networks, the global μ-stability of the delayed QVNNs was studied by constructing Lyapunov-Krasovskii functional. In [25], by virtue of Homeomorphic mapping theorem, Lyapunov theorem and inequality technique, the robust stability of QVNNs with time delays and parameter uncertainties was studied.

As we all know, the number of equilibrium points comes to play a crucial role in the practical applications of neural networks. For example, in optimization application, the networks are required to have only one monostable equilibrium point (see [26–30] and references therein). Nevertheless, some other applications require neural networks to have multistable equilibrium points, such as pattern recognition, digital selection, decision making and associative memory (see [11,31,32] and references therein). Therefore, it is of prime importance to investigate the multistability problem of the neural networks. For RVNNs or CVNNs, numerous results on the multistability have been investigated in [33–47]. In [33], by constructing parameter conditions and using inequality technique, it was verified that the delayed RVNNs with two classes of activation functions can have 3^n equilibria and 2^n of them are locally exponentially stable. In [35], it was proved that the n-dimensional RVNNs with nondecreasing piecewise linear activation functions with $2r$ corner points can have $(2r+1)^n$ equilibria and $(r+1)^n$ of them are locally exponentially stable. In [39], the multistability problem of CVNNs with real-imaginary-type activation functions was studied. It was found that the n-dimensional CVNNs can have $[(2\alpha+1)(2\beta+1)]^n$ $(\alpha, \beta \geq 1)$ equilibria and $[(\alpha+1)(\beta+1)]^n$ equilibria of them are locally exponentially stable. In [40], for a n-neuron delayed CVNNs with nondecreasing activation functions, sufficient criteria were achieved to assure the existence of 9^n equilibria and 4^n of them were locally exponentially stable. It is obvious that the number of the equilibria of CVNNs is much more than the ones of RVNNs under the same dimensions. This means that the storage capacity of the CVNNs is larger, which is valuable when the neural networks are applied in associative memory applications. So far, the multistability of the QVNNs hardly has gained attention, which drives us to develop the present research.

It is commonly known that the multistability analysis of neural networks heavily depends on the activation functions. Therefore, choosing the proper activation functions is particularly important for the QVNNs. In many previous literature, the activation functions of neural networks are nondecreasing and continuous [33–40,48]. Besides, some studies have obtained for the neural networks with discontinuous activations [42–46]. Heretofore, the multistability for QVNNs with nonmonotonic piecewise nonlinear activation functions has not been considered in the literature, which drives us to study the present research from another motivation.

In practical applications, time delays are inevitable factors in neural networks because of the finite switching speed of amplifiers and the interneurons conduction distances. The existence of time delays may become a source of instability or oscillation for the systems. Thus, the dynamics of the neural networks with time delays should be taken into account [49–53]. Besides, many studies about the chaos which can have an impact on inducing instability of systems have been investigated in [54–56] and references therein. In most previous literatures, the multistability analysis of the neural networks were developed with the bounded delays. As pointed in [57,58], time delays may be unbounded in some practical applications, which means that the entire history may affect the present. Therefore, it is significant to study the dynamics of neural networks with unbounded delays. Recently, much attention has been devoted into multiple μ-stability analysis of neural networks with unbounded time-varying delays [59–61], which is a generalization of multiple exponential stability, multiple power stability, multiple log-stability and multiple log–log-stability. To best of the authors' knowledge, there are few results proposed previously on the multiple μ-stability problem of the QVNNs.

Motivated by the above discussions, we address the problem of the multiple μ-stability analysis for the QVNNs with nonmonotonic piecewise nonlinear activation functions and unbounded time-varying delays in this paper. Some novel algebraic criteria are established to guarantee that the QVNNs can have 5^{4n} equilibrium points, and 3^{4n} of them are locally μ-stability. The novelties and contributions of the paper can be summarized as follows. (1) This paper extends the multiple μ-stability results of RVNNs and CVNNs to QVNNs, which can obtain even more equilibria compared with the previous studies and can achieve larger storage capacity in practical applications. (2) The obtained results in this paper are generalizations of the multiple exponential stability, multiple power stability, multiple log-stability, multiple log–log-stability. (3) The activation functions used in this paper are an extension to the ones used in [59].

The remaining of this paper is organized as follows. In Section 2, model description of the QVNNs and preliminaries are presented. In Section 3, the existence of stability of multiple equilibria is investigated for the QVNNs with unbounded time-varying delays and nonmonotonic piecewise nonlinear activation functions. In Section 4, one numerical simulation is given to clarify the theory and distinct dynamical behaviors for different activation functions. Finally, conclusions are drawn in Section 5.

Notations: The notations are quite standard. Throughout this paper, $\mathbb{R}^{m\times n}$, $\mathbb{C}^{m\times n}$, $\mathbb{Q}^{m\times n}$ denote, respectively, the set of $m\times n$ real-valued, complex-valued and quaternion-valued matrices. Function $h = h^R(t) + \mathrm{i}h^I(t) + \mathrm{j}h^J(t) + \mathrm{k}h^K(t)$ denotes quaternion-valued function, where $h^R(t)$, $h^I(t)$, $h^J(t)$, $h^K(t)$ are all real-valued functions and i, j, k obey Hamilton rules: