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Superconvergence of Discontinuous Galerkin Methods for Nonlinear Delay Differential Equations with Vanishing Delay

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Abstract. In this paper, we investigate the local superconvergence of the discontinuous Galerkin (DG) solutions on quasi-graded meshes for nonlinear delay differential equations with vanishing delay. It is shown that the optimal order of the DG solution at the mesh points is $O(h^{2m+1})$. By analyzing the supercloseness between the DG solution and the interpolation $\Pi_h u$ of the exact solution, we get the optimal order $O(h^{m+2})$ of the DG solution at characteristic points. [We then extend the convergence results of DG solutions to state dependent delay differential equations.](#) Numerical examples are provided to illustrate the theoretical results.

Keywords. *superconvergence, nonlinear delay differential equations, discontinuous Galerkin methods, vanishing delay, state dependent delay.*

1 Introduction

We consider the following nonlinear vanishing delay differential equation (DDE)

$$\begin{aligned} u'(t) &= f(t, u(t), u(\theta(t))), \quad t \in J = [0, T], \\ u(0) &= u_0, \end{aligned} \tag{1.1}$$

where the delay function θ is subject to the following conditions:

1. $\theta(0) = 0$ and $\theta(t) < t$ for $t > 0$,
2. $\min_{t \in J} \theta'(t) =: q_0 > 0$,

and the function $f \in C(J)$.

Delay differential equations are important mathematical models that describe various real-life phenomena such as biological, physical and chemical systems. The exact analytical solutions of the DDEs are not available in general, one has to rely on numerical methods to find approximate solutions. There are many numerical methods for DDEs including Runge-Kutta methods ([2, 19, 30]), linear multi-step methods [25], θ -methods [20],

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