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On the approximation of weakly plurifinely plurisubharmonic functions

Nguyen Xuan Hong[a,*], Hoang Van Can[b]

[a] *Department of Mathematics, Hanoi National University of Education, 136 Xuan Thuy Street, Cau Giay District, Hanoi, Viet Nam*

[b] *Department of Basis Sciences, University of Transport Technology, 54 Trieu Khuc, Thanh Xuan District, Hanoi, Viet Nam*

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Abstract

In this note, we study the approximation of singular plurifine plurisubharmonic function u defined on a plurifine domain Ω. Under some condition we prove that u can be approximated by an increasing sequence of plurisubharmonic functions defined on Euclidean neighborhoods of Ω.

Keywords: Complex variables; Plurifinely pluripotential theory; Plurifinely plurisubharmonic functions

1. Notation and main result

The plurifine topology $\mathcal{F}$ on a Euclidean open set D is the smallest topology that makes all plurisubharmonic functions on D continuous. Notions pertaining to the plurifine topology are indicated with the prefix $\mathcal{F}$ to distinguish them from notions pertaining to the Euclidean topology on $\mathbb{C}^n$. For a set $A \subset \mathbb{C}^n$ we write $\overline{A}$ for the closure of A in the one point compactification of $\mathbb{C}^n$, $\overline{A}^{\mathcal{F}}$ for the $\mathcal{F}$-closure of A and $\partial_{\mathcal{F}} A$ for the $\mathcal{F}$-boundary of A.

Let Ω be a bounded $\mathcal{F}$-domain in $\mathbb{C}^n$. A function $u : \Omega \to [-\infty, +\infty)$ is said to be $\mathcal{F}$-plurisubharmonic if u is $\mathcal{F}$-upper semicontinuous and for every complex line l in $\mathbb{C}^n$, the

* Corresponding author.
E-mail addresses: xuanhongdhsp@yahoo.com (N.X. Hong), vancan.hoangk4@gmail.com (H. Van Can).

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restriction of u to any $\mathcal{F}$-component of the finely open subset $l \cap \Omega$ of l is either finely subharmonic or $\equiv -\infty$. El Kadiri, Fuglede and Wiegerinck [10] proved the most important properties of the $\mathcal{F}$-plurisubharmonic functions. El Kadiri and Wiegerinck [12] defined the complex Monge–Ampère operator for finite $\mathcal{F}$-plurisubharmonic functions on an $\mathcal{F}$-domain Ω. Recently, Hong and coauthors have been successfully pushing the theory of $\mathcal{F}$-plurisubharmonic functions (see [16,17], [18,21]). The aim of this note is to study the conditions on u and Ω such that u can be approximated by an increasing sequence of plurisubharmonic functions defined on Euclidean neighborhoods of Ω.

When Ω is bounded Euclidean domain with $\mathcal{C}^1$-boundary, Fornæss and Wiegerinck [13] proved that if u is continuous on $\overline{\Omega}$ then u can be approximated uniformly on $\overline{\Omega}$ by a sequence of smooth plurisubharmonic functions defined on Euclidean neighborhoods of Ω.

When Ω is bounded hyperconvex domain, according to the results by [4,5,8,15] and other authors, the approximation is possible if the domain Ω has the $\mathcal{F}$-approximation property and u belongs to one of the Cegrell's classes in Ω.

When Ω is bounded $\mathcal{F}$-domain, the authors gave in [21] the kind of Ω and u that are in line with the $\mathcal{F}$-set up to make the approximation possible.

The purpose of this note is to extend the result of [21]. In analogy with the set up of the hyperconvex domain to make the approximation possible, we introduce the following:

Definition 1.1. Let Ω be a bounded $\mathcal{F}$-hyperconvex domain, i.e., it is a bounded, connected, and $\mathcal{F}$-open set such that there exists a negative bounded plurisubharmonic function γ_Ω defined in a bounded hyperconvex domain $\Omega' \supset \Omega$ such that $\Omega = \{\gamma_\Omega > -1\}$ and $-\gamma_\Omega$ is $\mathcal{F}$-plurisubharmonic in Ω. We say that Ω has the $\mathcal{F}$-approximation property if there exists an increasing sequence of negative plurisubharmonic functions ρ_j defined on bounded hyperconvex domains Ω_j such that $\Omega \subset \Omega_{j+1} \subset \Omega_j$ and $\rho_j \nearrow \rho \in \mathcal{E}_0(\Omega)$ a.e. on Ω as $j \nearrow +\infty$. Here

$$\mathcal{E}_0(\Omega) := \{u \in \mathcal{F}\text{-}PSH^-(\Omega) \cap L^\infty(\Omega) : \int_\Omega (dd^c u)^n < +\infty$$
$$\text{and } \forall \varepsilon > 0,\ \exists \delta > 0,\ \overline{\{u < -\varepsilon\}} \subset \{\gamma_\Omega > -1 + \delta\}\}.$$

Every bounded hyperconvex domain is $\mathcal{F}$-hyperconvex. Example 3.3 in [21] showed that there exists a bounded $\mathcal{F}$-hyperconvex domain Ω that has the $\mathcal{F}$-approximation property, moreover, it has no Euclidean interior point. For the precise definition and properties of the class $\mathcal{F}(\Omega)$ we refer the reader to the next section. Our main result is the following theorem.

Theorem 1.2. *Let Ω be a bounded $\mathcal{F}$-hyperconvex domain and let $u \in \mathcal{F}(\Omega)$. Assume that Ω has the $\mathcal{F}$-approximation property. Then there exists an increasing sequence of plurisubharmonic functions u_j defined on Euclidean neighborhoods of Ω such that $u_j \nearrow u$ a.e. on Ω as $j \nearrow +\infty$.*

The note is organized as follows. In Section 2, we introduce and investigate the class $\mathcal{F}(\Omega)$. Section 3 is devoted to prove Theorem 1.2.

2. The class $\mathcal{F}(\Omega)$

Some elements of pluripotential theory (plurifine potential theory) that will be used throughout the paper can be found in [1–22]. We denote by $\mathcal{F}\text{-}PSH^-(\Omega)$ the set of negative $\mathcal{F}$-plurisubharmonic functions defined in an $\mathcal{F}$-open set Ω. First, we recall the definition of the complex Monge–Ampère measure for finite $\mathcal{F}$-plurisubharmonic functions.