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Interpolating sequences in mean

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Abstract

This paper deals with interpolating sequences $(z_n)_n$ for two spaces of holomorphic functions f in the unit disk $\mathbb{D}$ in $\mathbb{C}$: those that are bounded and those that satisfy a Lipschitz condition $|f(z) - f(w)| \leq c\,|z-w|^\alpha$, $0 < \alpha \leq 1$. Given a sequence of values $(w_n)_n$ in a certain target space, we look for a function f interpolating 'in mean", that is, with $(f(z_1) + \cdots + f(z_n))/n = w_n$, $n \geq 1$. We obtain target spaces when we prescribe that the corresponding interpolating sequences be the uniformly separated ones or the union of two uniformly separated ones.

Keywords: Interpolating sequence; Uniformly separated sequence; Bounded holomorphic function; Lipschitz class

1. Introduction and statement of the problems

Let H^p $(0 < p < \infty)$ be the Hardy space of holomorphic functions f in $\mathbb{D}$ such that

$$\sup_{0 \leq r < 1} \int_{-\pi}^{\pi} |f(re^{i\theta})|^p \, d\theta < \infty$$

and let H^∞ denote the space of those such that $\|f\|_\infty = \sup_{z\in\mathbb{D}}|f(z)| < \infty$. The space BMOA consists of those functions in H^1 whose boundary values have bounded mean oscillation on $\partial\mathbb{D}$ and it is closely related to the Hardy spaces ($H^\infty \subset \text{BMOA} \subset \bigcap_{p>0} H^p$). For $0 < \alpha \leq 1$, let Λ_α be the Lipschitz class of order α consisting of the holomorphic functions f in $\mathbb{D}$, continuous on

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$\overline{\mathbb{D}}$ and verifying

$$\|f\|_{\Lambda_\alpha} = \sup_{z\in\overline{\mathbb{D}}}|f(z)| + \sup_{z,w\in\mathbb{D}} \frac{|f(z)-f(w)|}{|z-w|^\alpha} < \infty.$$

We write $Z = (z_n)_n$ for any sequence of different points in $\mathbb{D}$ satisfying the Blaschke condition $\sum_n(1-|z_n|) < \infty$ and put $W = (w_n)_n$ for sequences in $\mathbb{C}$. Let l^p denote the space of sequences W such that $\sum_n|w_n|^p < \infty$ and let l^∞ be the space of those such that $\|W\|_\infty = \sup_n|w_n| < \infty$. Let $l_\alpha(Z)$ denote the space consisting of the sequences W such that

$$\|W\|_{l_\alpha(Z)} = \sup_n|w_n| + \sup_{i,\,j\in\mathbb{N}} \frac{|w_i - w_j|}{|z_i - z_j|^\alpha} < \infty.$$

Let B be the Blaschke product with zeros at Z, that is, the function in H^∞ defined by

$$B(z) = \prod_{n=1}^{\infty} \frac{|z_n|}{z_n}\frac{z_n - z}{1-\overline{z}_n z},$$

and let $B_{i_1,\ldots,i_k}$ denote the Blaschke product with zeros at $Z \setminus \{z_{i_1}, \ldots, z_{i_k}\}$. We put ρ for the pseudohyperbolic metric in $\mathbb{D}$, that is,

$$\rho(z, w) = |(z-w)/(1-\overline{z}w)|.$$

As usual, we write c for strictly positive constants that may change from one occurrence to the next one. Recall that given $\tau \in \partial\mathbb{D}$ and $\mu \in [1, \infty)$, the set

$$\{z \in \mathbb{D} : |1-\overline{\tau}z| \le \mu(1-|z|)\}$$

is called a *Stolz angle with vertex at* τ. Recall also that Z is said *separated* if $\inf_{m\neq n}\rho(z_m, z_n) > 0$ and Z is called *uniformly separated* if $|B_m(z_m)| \ge c$ for all $m \in \mathbb{N}$. Clearly, every uniformly separated sequence is separated; reciprocally, every separated sequence in a Stolz angle is uniformly separated [11].

Let X be a Banach space of holomorphic functions in $\mathbb{D}$ and let T be a *target space* of sequences in $\mathbb{C}$. Let $R^Z : X \to T$ denote the restriction operator on Z, that is, $R^Z(f) = (f(z_n))_n$. The classical interpolation problems consist in characterizing the so-called "interpolating sequences":

Definition 1.1. A sequence Z is called interpolating for (X, T) if $R^Z(X) = T$.

In some mathematical topics it makes sense to consider averages of the objects that are studied in order to relate a certain behavior of them with that of their averages; such is the case, for example, of the sequences of numbers, random variables..., when we examine their convergence. Since that is possible and natural in the context of complex interpolation, in this paper we introduce averages of values of holomorphic functions in $\mathbb{D}$ (objects) to study their interpolating properties (behavior).

We define the average operator $S^Z : X \to T$ by $S^Z(f) = (S_n^Z(f))_n$, where

$$S_n^Z(f) = \frac{f(z_1) + \cdots + f(z_n)}{n}, \quad n \ge 1.$$

Unlike R^Z, the operator S^Z has memory (for each n, all values from $f(z_1)$ to $f(z_n)$ are taken into account), which gives it a peculiar interest. Note that S^Z depends on the numbering order of the points in Z, so that it does not adapt automatically to rearrangements of Z. We introduce the following sequences: