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Convolution factorability of bilinear maps and integral representations

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Abstract

In this paper we consider a special class of continuous bilinear operators acting in a product of Banach algebras of integrable functions with convolution product. In the literature, these bilinear operators are called 'zero product preserving', and they may be considered as a generalization of Lamperti operators. We prove a factorization theorem for this class, which establishes that each zero product preserving bilinear operator factors through a subalgebra of absolutely integrable functions. We obtain also compactness and summability properties for these operators under the assumption of some classical properties for the range spaces, as the Dunford–Pettis property or the Schur property and we give integral representations by some concavity properties of operators. Finally, we give some applications for integral transforms, and an integral representation for Hilbert–Schmidt operators.

Keywords: Bilinear operators; Factorization; Zero product preserving map; Summability; Hilbert–Schmidt operators; Integral representation

1. Introduction

The recent paper [15] presents a class of bilinear operators B acting in the Cartesian product of the space $L^2(\mathbb{T})$ by itself –$\mathbb{T}$ is the real line mod 2π – with the following property: B is zero

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valued for the couples of functions $f, g \in L^2(\mathbb{T})$ that are convolution-null, that is, that satisfies $f * g = 0$.

The investigation of bilinear operators having this property leads in a natural way to consider a particular class of linear operators, called *Lamperti operators* — which are also known as *disjointness preserving* or *separating maps*. They are operators between vector lattices that transform disjoint elements into disjoint elements (for more information, see [1]). The notion of Lamperti operator has been brought to the framework of the function spaces and the Banach algebras; the term 'zero product preserving' is commonly used for these operators (see [2,4,5] and references therein). Separating maps are connected to zero-product preserving bilinear maps. Indeed, if we consider the bilinear map $\psi : A \times A \to B$ defined by $\psi(a, b) = S(a)S(b)$, where S is a linear operator from the Banach algebra A to B, it is clear that ψ is zero product preserving – that is, it is 0-valued for couples with a product equal to zero –, whenever the map S is separating. Bilinear operators having this and related properties have been studied by some authors in the last years. Recently, Alaminos et al. [2] have analyzed linear surjective operators $T : A \to B$ from a Banach algebra A to another Banach algebra B with the property that $ab = 0$ implies $T(a)T(b) = 0$ for $a, b \in A$. To investigate the class of zero product preserving operators, they have introduced bilinear continuous operators $\phi : A \times A \to B$ defined by $\phi(a, b) = T(a)T(b)$. It is clear that $\phi(a, b) = 0$ whenever $ab = 0$ by the zero product preserving property of the operator T. They have obtained a class of Banach algebras A that satisfy the equality $\phi(ab, c) = \phi(a, bc)$ $(a, b, c \in A)$ for every continuous zero product preserving bilinear map $\phi : A \times A \to B$. By adding some conditions to the algebra, they have proved that the requirement $\phi(ab, c) = \phi(a, bc)$ gives a factorization for the bilinear operator ϕ as $\phi(a, b) := P(ab)$ for a certain linear map $P : A \to B$. They obtained that a zero product preserving continuous bilinear operator defined on the product of a Banach algebra with an approximate identity has such a linear factorization under their assumptions [2].

On the other hand, Buskes and van Rooij first considered the factorization of zero product preserving bilinear maps defined on vector lattices termed *orthosymmetric operator* [8]. It is useful to note that in this case, two functions are said to be disjoint if and only if they have 0-valued pointwise product (μ-a.e.). For an order bounded orthosymmetric bilinear map $T : E \times E \to F$, where E and F are the Archimedean vector lattices, they showed that it satisfies the symmetry condition $T(x, y) = T(y, x)$ if it is positive — that is, $T(x, y) \geq 0$ if $x \geq 0$, $y \geq 0$. As a result of this theorem, it is seen that a bimorphism $B : E \times E \to F$ is symmetric if and only if it has the orthosymmetry condition. The same authors used the orthosymmetric bilinear operators for obtaining the squares of vector lattices (see [9]). A recent study related to orthosymmetric maps was given by Ben Amor. He investigated the commutators of orthosymmetric bilinear maps and gave a generalized class of orthosymmetric bilinear maps related to the symmetry condition by the concept of relatively uniform continuity [6].

Recently, Alaminos et al. [3] have studied the orthogonality preserving linear maps defined on the Banach algebra $L^1(G)$ into $L^1(H)$ in an algebraic manner. The term 'orthogonality preserving linear maps' is another name for the Lamperti operators that is sometimes used in the literature. In this paper, they investigate the linear operator T such that $f * g = 0$ implies $T(f) * T(g) = 0$.

Motivated by these ideas we shall introduce the notion of $*$-factorability for the bilinear operators defined on the product of the Banach algebra $L^1(\mathbb{T})$ and some of its subalgebras which are a $L^1(\mathbb{T})$-module. We center our attention on bilinear operators B

$$B(f, g) = 0 \text{ whenever } f * g = 0,$$

having a factorization through the convolution product $*$.

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